EFFICIENT AND LOW-COST NODE SEISMIC DATA RECOVERY BASED ON CURVELET COMPRESSION SENSING

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ABSTRACT

Owing to increasingly complex surface conditions of land seismic acquisition in petroleum exploration, conventional wired acquisition has become environmentally unfeasible, leading to the dramatic increase in exploration cost. We recover and reconstruct the acquired OBN data by the technology of compressed sensing based on curvelet transform, iterative threshold and sampling matrix according to the acquired data. As per model tests, data restoration is satisfactory in the context of complete data on both sides of missing traces and missing traces less than 6% of total traces. The threshold and curvelet scale will be defined depending on measured data. A field OBN data application with 1378 shots yields good results of missing data restoration and reconstruction. The improved event continuity and information content demonstrate the validity of compressed sensing for data restoration and reconstruction.

KEY WORDS: compressed sensing; data restoration and reconstruction; OBN data; curvelet transform

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INTRODUCTION

Owing to increasingly complex surface conditions of land seismic acquisition in petroleum exploration, conventional wired acquisition has become environmentally unfeasible, leading to the dramatic increase in exploration cost and security risks of field operation. These challenges have made oil companies and service contractors flinch. Wireless node acquisition developed in the latest years based on some key techniques, such as storage technology, battery life, and time correction, is much superior to wired acquisition in surface environmental feasibility and operational flexibility and thus makes seismic exploration come true in the prospects with complicated surface conditions.

For ocean bottom node (OBN) technique, separate geophones are positioned at sea floor for data recording. After excitation, data acquired by geophones will be exported for further processing and interpretation. OBN technique is not subject to transmission cables and thus more flexible than ocean bottom cable (OBC) technique. Besides, OBN technique is less affected by sea water and can obtain high-quality data because of the advantages of continuous recording, multiple components, and high signal-to-noise ratio. This technique is significant to improved seismic imaging and reservoir monitoring [1] and has been widely used in reservoir monitoring and other activities. Shell used OBN for reservoir monitoring in Mars field in the Gulf of Mexico in 2004 [2] and in Bonga deepwater field in Nigeria in 2008, obtaining data of higher quality than streamer acquisition [3]. Seabed Geosolution researched into the equipment of automatic OBN acquisition. Shell developed a new generation of OBN equipment (Flying Node) and solved the problems of slow landing and picking node facilities with low accuracy by undersea teleoperators [4]. However, wireless node acquisition cannot be monitored in the process of operation, leading to seismic data gaps caused by abnormal performance of some nodes and even non-performance of continuous nodes. In serious cases, it is necessary to shoot again to reacquire missing data, which decreases the efficiency of exploration and increases the workload and cost of field seismic acquisition. To solve the problem of data missing in node acquisition, increase the efficiency of exploration, and decrease the workload of field acquisition and the cost of exploration, it is urgent to develop the technique of seismic data restoration and reconstruction to replace the process of field reshooting and re-acquirement technically.

Seismic data reconstruction is to technically recover the gaps in field seismic data at defined sampling rate and enrich seismic data with sufficient geophysical information to support geophysical exploration and development. There are three routine methods: filter, wave field operator, and transform domain, the sampling rate of which should be large enough to satisfy the requirement of the Nysuist sampling theorem; hence, large storage space is needed. To break through the limitation of routine methods, compressed sensing was developed [5-7] to encode original signals using the frequencies far below the Nyquist frequency and then restore signals accurately or reconstruct signals approximately with small errors using a reconstruction algorithm. As to compressed sensing essentially, if the signals are sparse in an orthogonal transform domain and a measurement matrix is defined to make the orthogonal bases of its sparse transform uncorrelated, the signals will be projected from a higher space to a low-dimensional space using this matrix; high frequencies can be restored and reconstructed using a recovery algorithm [8]. This process includes three steps: sparse representation of signals, measurement matrix (sampling method) definition, and reconstruction algorithm design.

Sparse representation of signals can be accomplished through Fourier transform, wavelet transform, curvelet transform, etc. Fourier transform realizes signal analysis in the frequency domain based on the interconversion between time domain and frequency domain, but it is a global transformation in the whole time domain and thus cannot characterize the frequency spectrum at local time [9]. To localize the sudden changes of signals, Gabor (1946) formulated the short-time Fourier transform [10], which uses a window function for local partition of signals and Fourier analysis to identify local frequencies. The window size and geometry are constant, and thus it is unfeasible for complex seismic data owing to great waveform variation with time [11]. For wavelet transform, the geometry of the window function is varied; hence, time resolution can be tuned according to frequency [12-14]. However, despite its wide application to seismic prospecting [15-18], it is still not a good choice for seismic data characterization. Curvelet transform uses curvilinear transform basis [19-20] to identify curves at different scales and in different directions [21]. As an optimal solution to the sparse representation of seismic data [22], it has been extensively applied to seismic data processing [23], including noise reduction [24-27], data reconstruction [28-31], and wave field simulation [32-33]. The algorithm of data reconstruction based on curvelet transform was created by integrating curvelet transform with compressed sensing [34].

A conventional regular sampling method adopts uniform sampling with equal interval. Sampling rate should conform to the Nysuist sampling theorem; otherwise, aliasing will lead to serious problems in seismic data restoration. An alternative is random undersampling, which can mitigate aliasing by converting aliases into low-amplitude noises and then filtering them out. Consequently, it is possible to accomplish data restoration and reconstruction at the frequencies below the Nyquist frequency. Gaussian random sampling, a commonly used method in compressed sensing, suffers from the problem that the spacing interval between missing traces is beyond control; thus, seismic data cannot be recovered because many continuous traces may be lost [35]. Compared with Gaussian random sampling, Jitter sampling divides the zone to be processed into many subzones, in each of which forced random sampling is performed at each point to control the spacing interval between two neighboring missing traces to a great extent for compressed sampled data reconstruction [36]. Hennenfent and Herrmann (2007) introduced 1D sampling into curvelet transform-based compressed sampling [26]. Tang (2010) generalized the sampling from one dimension to two dimensions and obtained good results of data restoration [37].

Reconstruction algorithm focuses on how to reconstruct complete data accurately using the sparse representation of signals; it is essentially a convex optimization problem to minimize L1 norm. A common method is basis pursuit (BP) algorithm, which searches for the most matched atom in the compressed sensing matrix in each iteration [38]. BP algorithm can be realized using interior point method, which is time consuming [39-40] despite accurate reconstruction of signals. Another common algorithm for convex optimization is iterative shrinkage-thresholding, which is simple but slow in convergence rate [41]. Matching pursuit (MP) algorithm searches for the atom in the sensing matrix matching most with current residual vector in each iteration [42], but it requires a large number of iterations. Orthogonal matching pursuit (OMP) algorithm [43] reduces the number of iterations by orthogonalizing the atom set selected by MP. OMP algorithm is much faster in convergence rate than convex optimization algorithm, and the results of data reconstruction are good.

Our efforts focus on three steps: sparse representation, sampling, and signal reconstruction, in compressed sensing, including different methods involved in each step and the application of compressed sensing to field data processing.

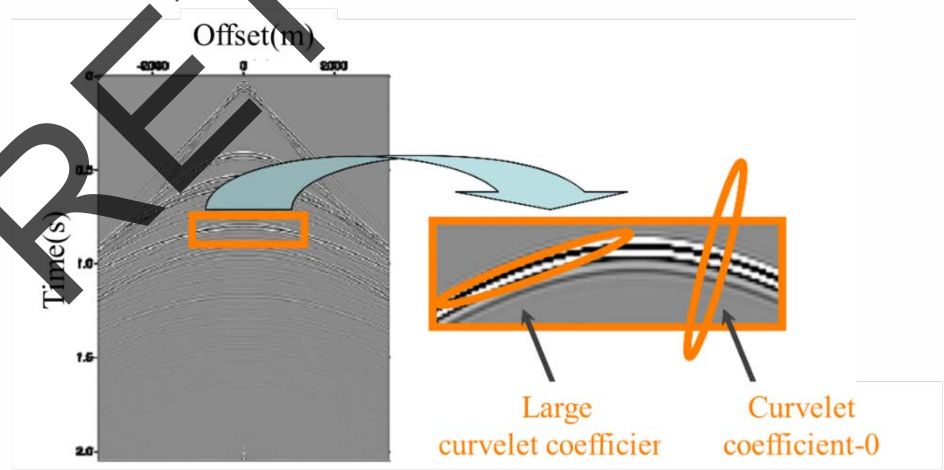
METHODOLOGY

Compressed sensing samples the sparse representation of signals using a measurement matrix, and then recovers and reconstructs the original high-frequency signals through solving an optimization problem. There are two premises; one is the sparsity of signals, and the other is that the sampling matrix is uncorrelated with the basis of sparse representation. Compressed sensing mainly includes three steps: sparse representation of signals, measurement matrix design, and signal restoration and reconstruction.

Sparse representation of signals

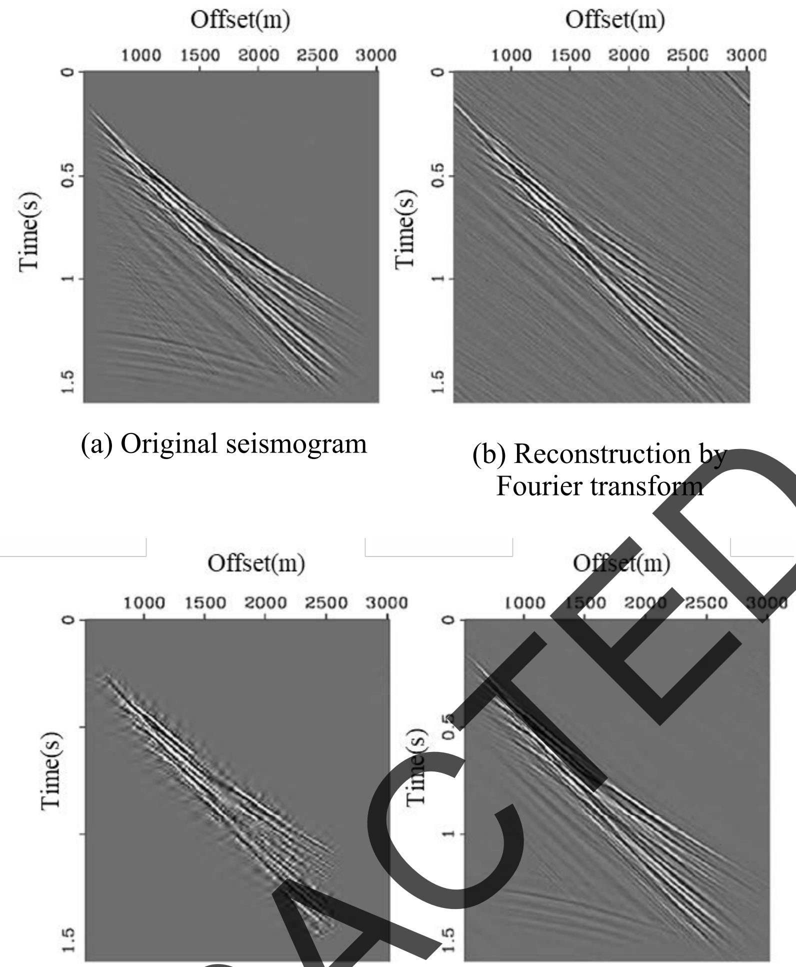
Sparse representation of signals is to brief signal representation by compressing signals and extracting key sparsities. Commonly used methods include Fourier transform, short-time Fourier transform, wavelet transform, ridgelet transform, and curvelet transform. Fourier transform is a global time-frequency analysis method, which deals with the overall features of signals in the time domain and cannot effectively identify local features. On the basis of Fourier transform, short-time Fourier transform divides the signals into a number of segments in the time domain by using a window function and then performs Fourier transform for each segment so as to characterize local features. The problem is that constant window size and geometry indicate a single resolution. Wavelet transform uses a damping wavelet basis with finite length, instead of an infinite basis of trigonometric function used in Fourier transform, to identify local features, which are restricted to point singularities. With respect to the issue of linear singularity, ridgelet transform converts linear singularities into point singularities through Radon transform and then uses point singularities identified by wavelet transform to capture linear singularities of signals. This method of linear singularity characterization is unfeasible for seismic signals with curved events. Based on wavelet transform and ridgelet transform, curvelet transform introduces an orientation parameter to accomplish optimum non-linear approximation of seismic data. Owing to multi-scale, multi-direction, and anisotropic properties, curvelet transform has been widely applied to seismic data restoration and reconstruction.

Figure 1 illustrates seismic wavefront approximated using a curvelet basis function, and Figure 2 shows the original seismic data in (a) and their reconstruction results by using Fourier transform in (b), wavelet transform in (c) and curvelet transform in (d) [15]. As shown in Figure 1, the curvelet coefficient is large when the curvelet basis parallels seismic wavefront; the coefficient decreases as the basis deviates from the direction of wavefront; the coefficient is at the minimum and close to zero when the basis is perpendicular to seismic wavefront. Hence, this part of data can be represented sparsely using a few large coefficients in parallel with seismic wavefront. Such behavior can be illustrated by reconstructing the 1% largest coefficients using different methods, as shown in Figure 2. Reconstruction by Fourier transform exhibits serious interference and missing data (Figure 2(b)). Reconstruction by wavelet transform does not suffer from the problem of interference, but wavefront texture with linear singularities cannot be effectively captured (Figure 2(c)). Both problems do not exist in the reconstruction result by curvelet transform (Figure 2(d)). Thus, we used curvelet basis for sparse representation in our study.



The orange box indicates an enlarged view. The orange ellipses represent curvelet basis.

Figure 1 Diagram of approximating seismic wavefront by using curvelet basis function



(a) Original seismogram (b) Reconstruction by Fourier transform

(c) Reconstruction by wavelet transform (d) Reconstruction by curvelet transform

Figure 2 Reconstruction results using different transforms

Measurement matrix design

Compressed sensing solves an underdetermined problem, which recovers and reconstructs the signals N with the length of N from the measured data y with the length of M ($M < N$) provided that the signals are k-sparse and the measurement matrix Ψ is uncorrelated with the sparse matrix Ψ [44]. If Φ is given, the sensing matrix $\Theta = \Phi\Psi$ will be obtained based on the designed sampling matrix Φ.

Methods

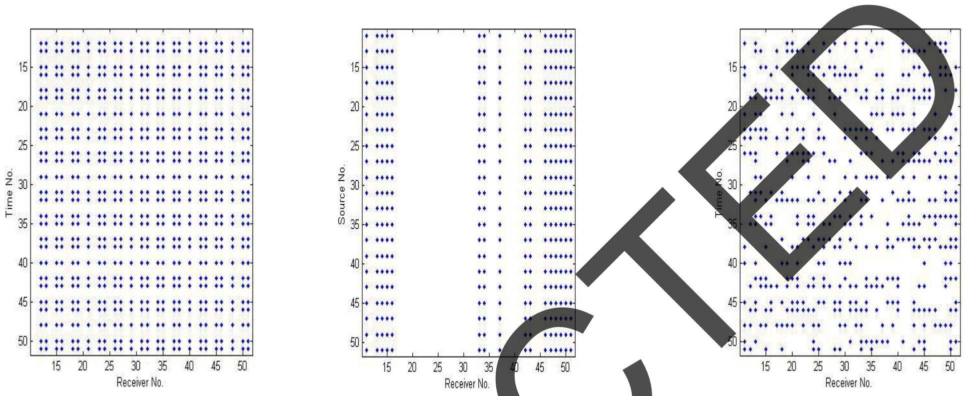
The sampling matrix R is designed in terms of sampling method. As per developmental sequence, sampling methods are classified as regular sampling and random sampling.

Regular sampling

A regular sampling method adopts uniform sampling with equal interval. If the Nysuist sampling theorem is satisfied, complete signals will be reconstructed without frequency aliasing. If the sampling frequency is below the Nyquist frequency, aliasing will happen; thus, it is impossible to reconstruct original signals accurately.

Random sampling

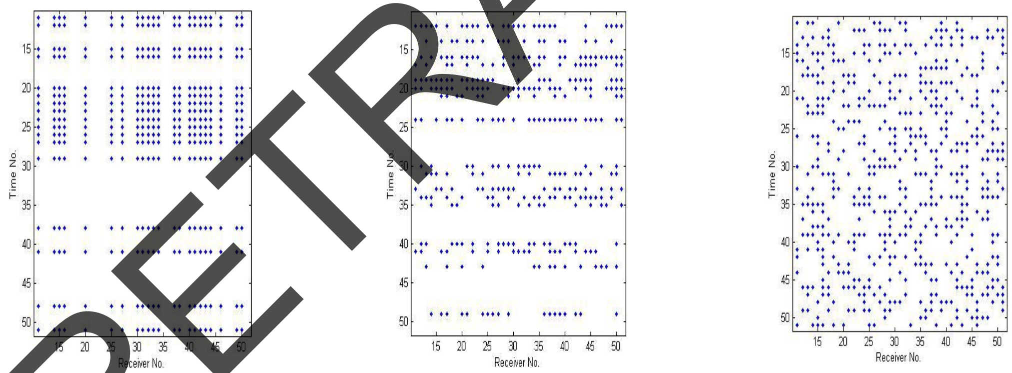
Random sampling is performed at heterogeneous intervals between sampling points. Unsampled points are uncorrelated with each other; hence, aliasing can be alleviated and eliminated effectively. Figure 3 illustrates several associations of sampling methods, where the lateral axis denotes the direction of receiver points and the vertical axis denotes the direction of shot points.



(a) Vertical regular sampling and lateral regular sampling

(b) Vertical regular sampling and lateral random sampling in the same way

(c) Vertical regular sampling and lateral random sampling in different ways



(d) Vertical random sampling in the same way and lateral random sampling in the same way

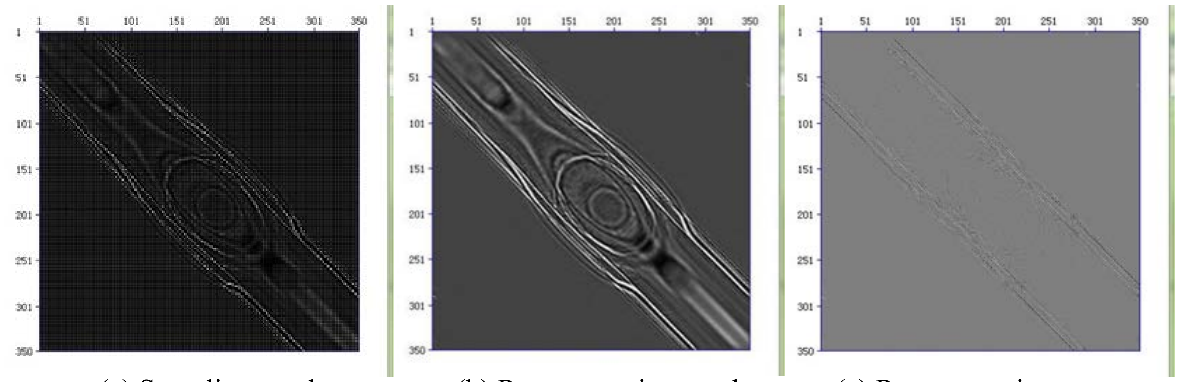
(e) Vertical random sampling in the same way and lateral random sampling in different ways

(f) Vertical random sampling in different ways and lateral random sampling in different ways

Figure 3 Diagrams of two-dimensional sampling methods

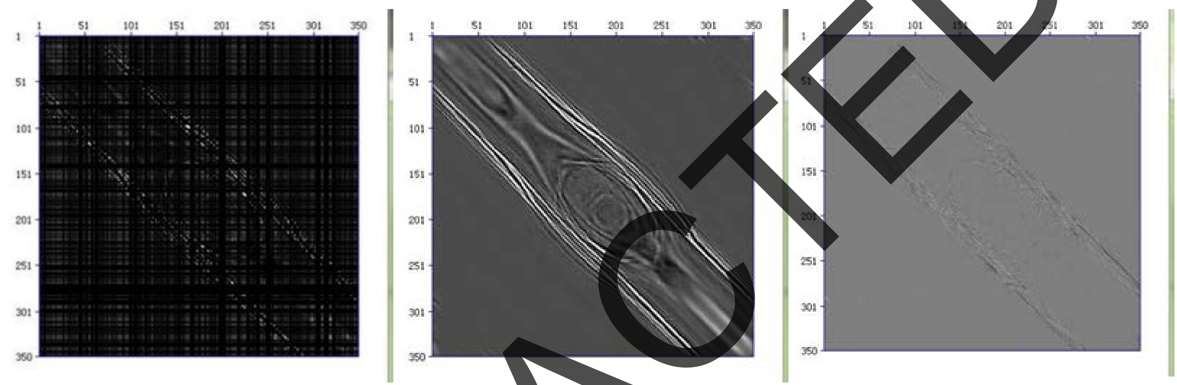
Method comparison

Figures 4 and 7 show the reconstruction results through curvelet-based compressed sensing for regular sampling and random sampling. The result of reconstruction improves with increasing irregularity and incoherence of sampling points.



(a) Sampling result (b) Reconstruction result (c) Reconstruction error

Figure 4 Reconstruction results for curvelet transform with regular sampling method



(a) Sampling result (b) Reconstruction result (c) Reconstruction error

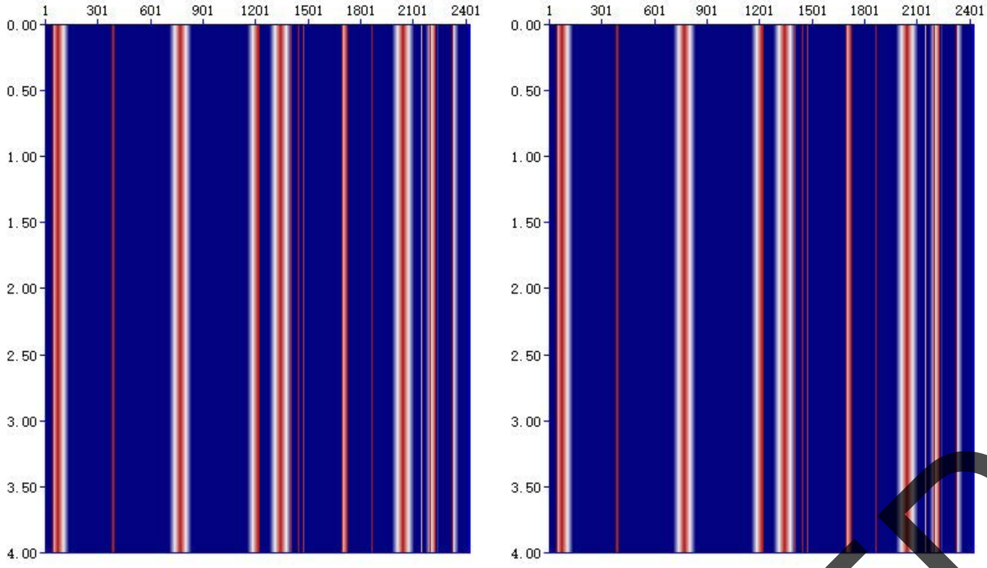
Figure 5 Reconstruction results for curvelet transform with random sampling method

APPLICATION

Sampling matrix construction

In the program implementation of seismic data processing, the sampling matrix is established in accordance with the distribution of sampling points for real data. This is different from the way of processing for theoretical data. For theoretical data, which are assumed to be complete, different sampling rates and sampling methods are used to gap and sample data, followed by data restoration. For real seismic data, the sampling method may be designed in accordance with data distribution. How to construct the sampling matrix is dependent on missing traces in original data.

Figure 6 shows the process of construction. A sampling matrix is designed to be equal in size to original data (Figure 6(a)), in which the sampling value is equal to 1 at the point with measured data and to 0 at the point with no measured data. Hence, a sampling matrix corresponding to original data is established (Figure 6(b)). Original data are equivalent to the sampling results of complete seismic data using the sampling matrix. After that, real seismic data will be recovered and reconstructed.



(a) Real OBN data (b) Sampling matrix established in accordance with real data

Figure 6 Construction of sampling matrix

Model tests

For effective data restoration and reconstruction, four parameters were tested using modelled data. The parameters are: threshold, data integrity on both sides of missing traces in the time domain, number of missing traces, and curvelet scale.

Threshold

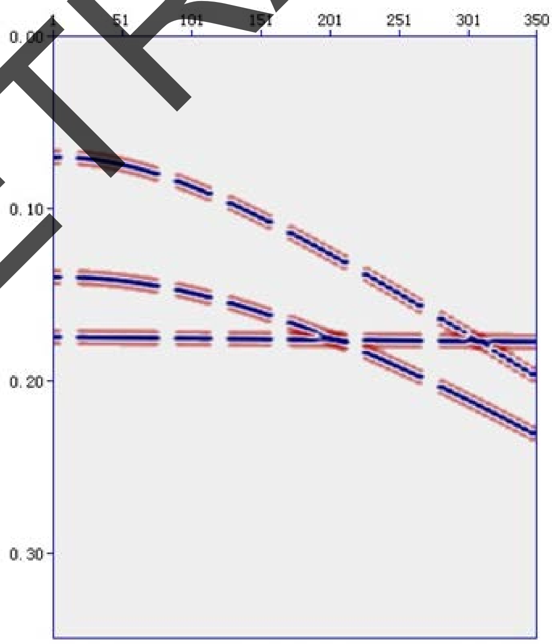
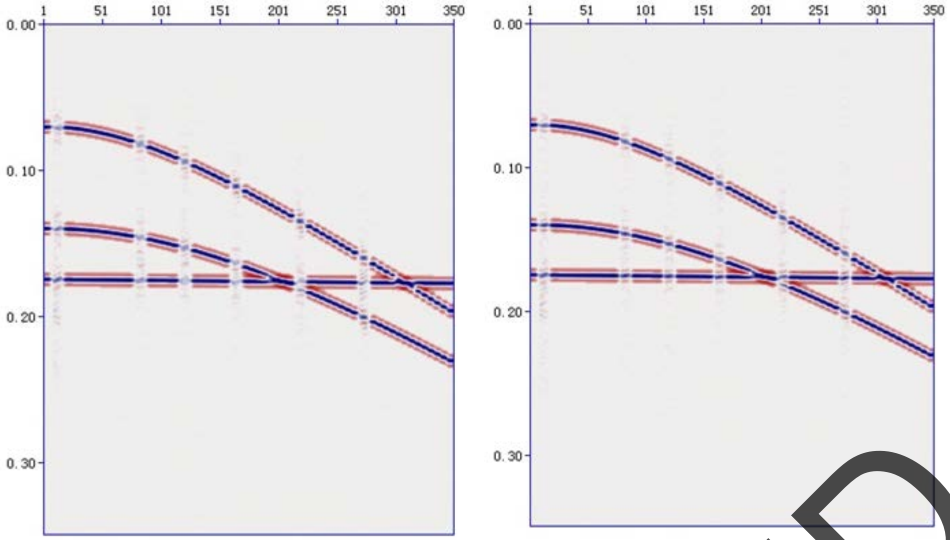
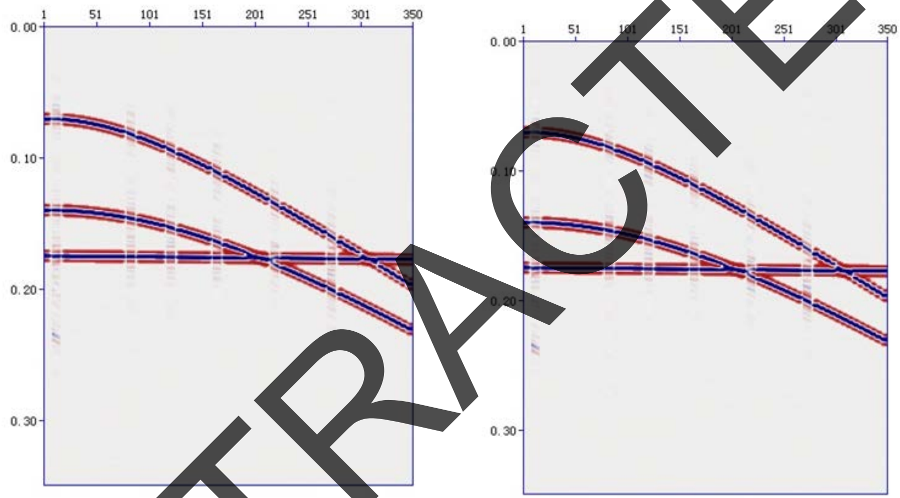


Figure 7 Testing data for threshold analysis



(a) Threshold=0.2 (b) Threshold=2



(c) Threshold=10 (d) Threshold=20

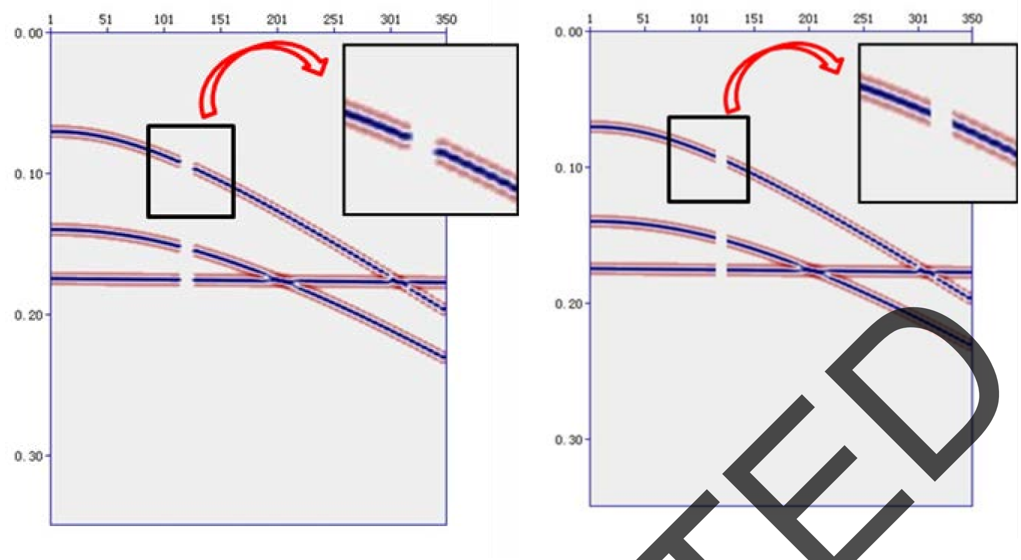
Figure 8 Reconstruction results of curvelet transform with different thresholds

The reconstructed data agree well with original data in curve shapes as the threshold increases; but a large threshold may lead to a blurred image. This means that there is an optimal threshold or threshold range in data processing.

The following part deals with original data integrity on both sides of missing traces in the time domain to discuss why there are gaps in reconstructed data.

Influence of data integrity

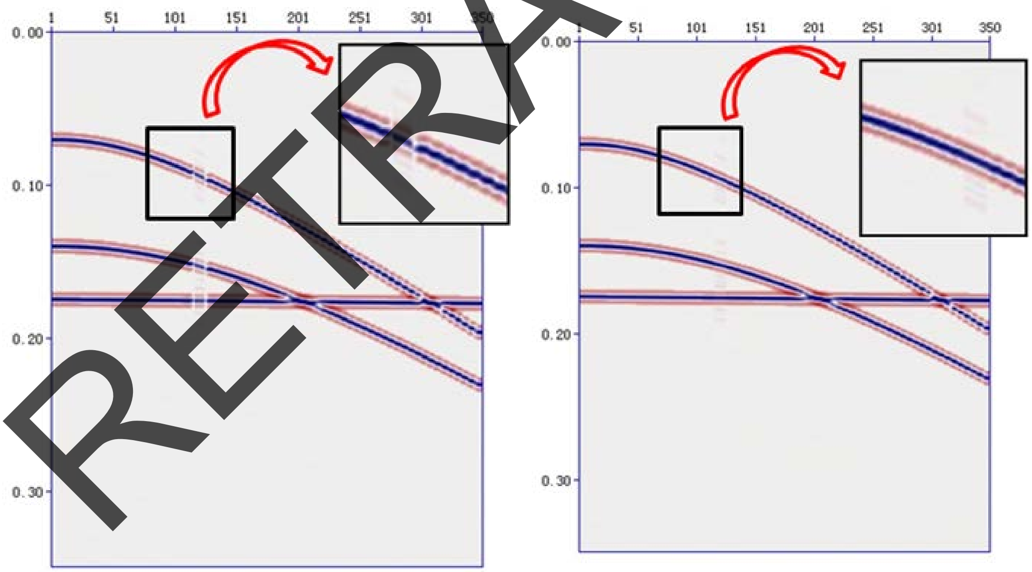
If original data are incomplete on both sides of the missing trace in the time domain (Figure 9(a)), these incomplete traces labelled as effective traces in the measurement matrix will lead to the gaps in recovered data (Figure 10(a)), which represent missing information in original data. If original data are complete without null values on both sides of the missing trace (Figure 9(b)), data restoration will yield good results (Figure 10(b)). In summary, original data integrity on both sides of missing traces in the time domain is a prerequisite to data restoration with good results.



(a) Incomplete data on both sides of the missing trace

(b) Complete data on both sides of the missing trace

Figure 9 Testing data for the integrity analysis on both sides of the missing trace

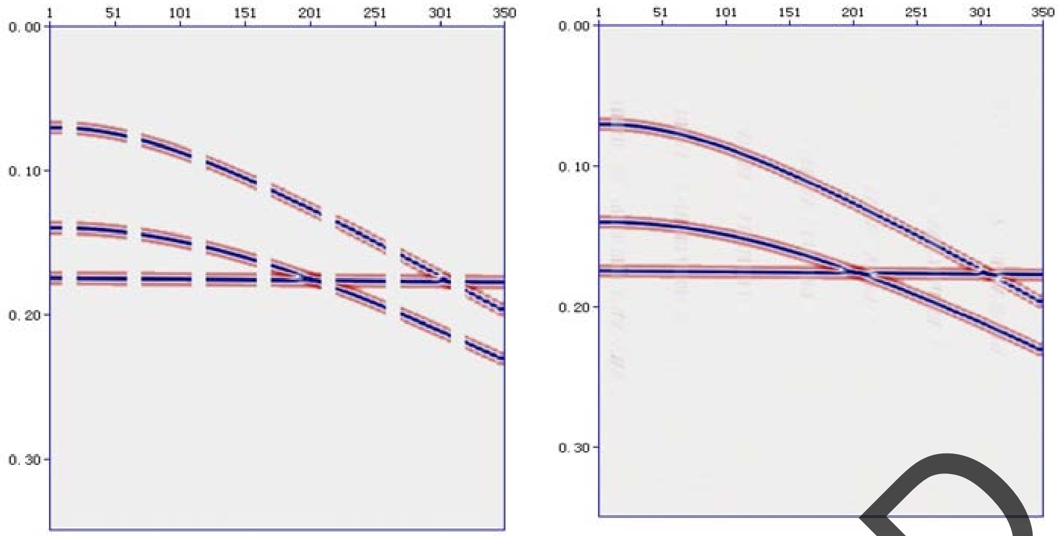


(a) Reconstruction of incomplete data

(b) Reconstruction of complete data

Figure 10 Reconstruction results of curvelet transform

Another test with several missing traces (Figure 11(a)) further illustrates good results of data restoration (Figure 11(b)).



(a) Seismic data with several missing traces

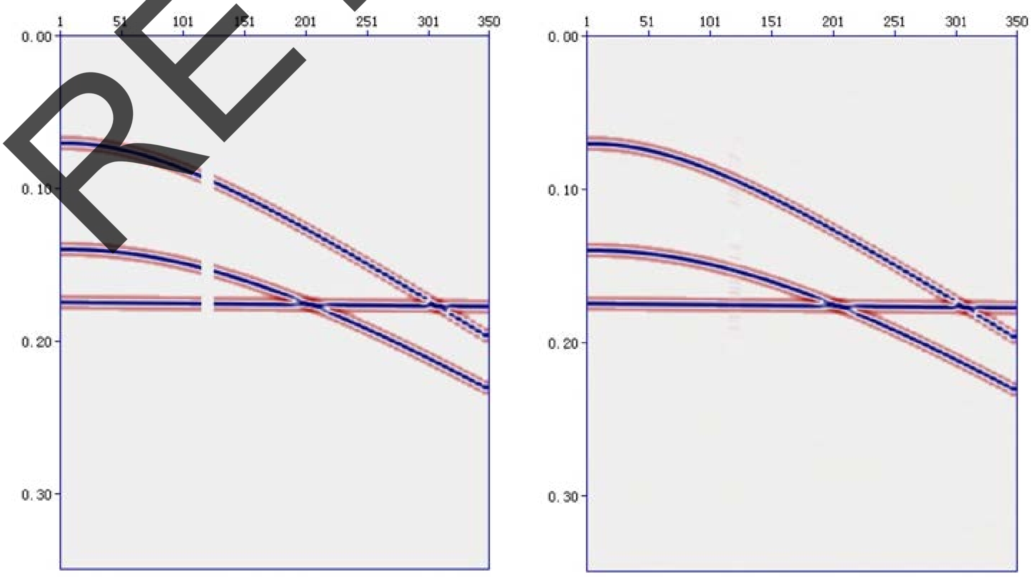
(b) Reconstruction results using curvelet transform

Figure 11 Reconstruction results of seismic data with several missing traces

Influence of number of missing traces

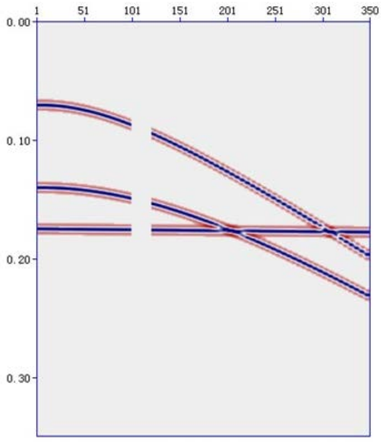
The following part deals with the number of missing traces in the context of complete data on both sides of missing traces (Figure 12).

Different numbers of missing traces were tested. For a data size of 350×350, restoration results are good for the number below 20, acceptable for the number of 30, and unsatisfactory for the number of 40. This means that the scale of curvelet basis is restrictive. In other words, restoration results are good for the curvelet scale of 6 and missing traces less than 6%.

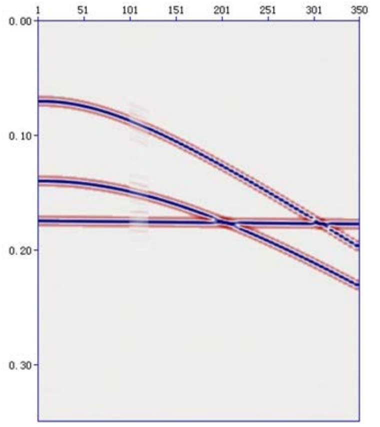


(a) 10 missing traces

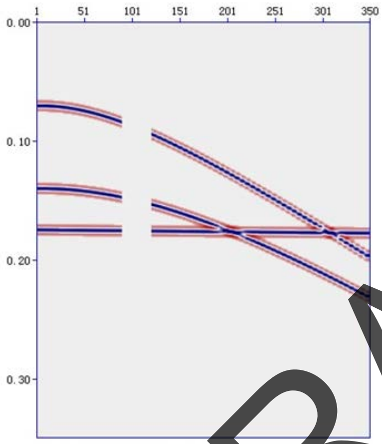
(b) Reconstruction results for 10 missing traces



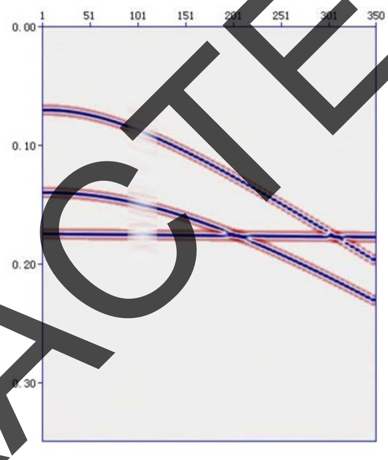
(c) 20 missing traces



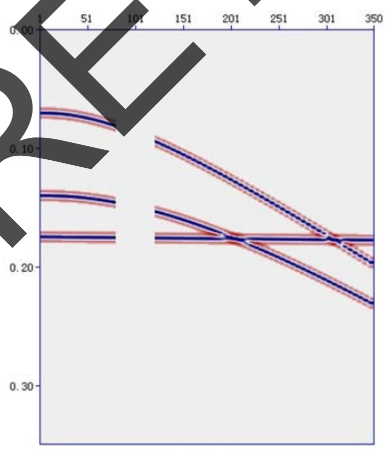
(d) Reconstruction results for 20 missing traces



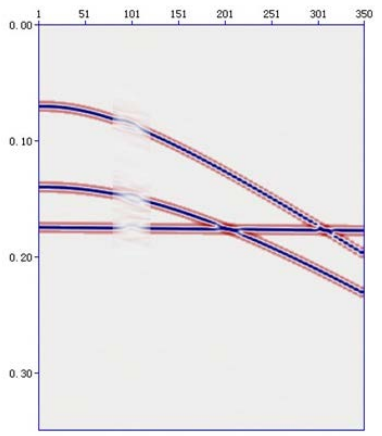
(e) 30 missing traces



(f) Reconstruction results for 30 missing traces



(g) 40 missing traces

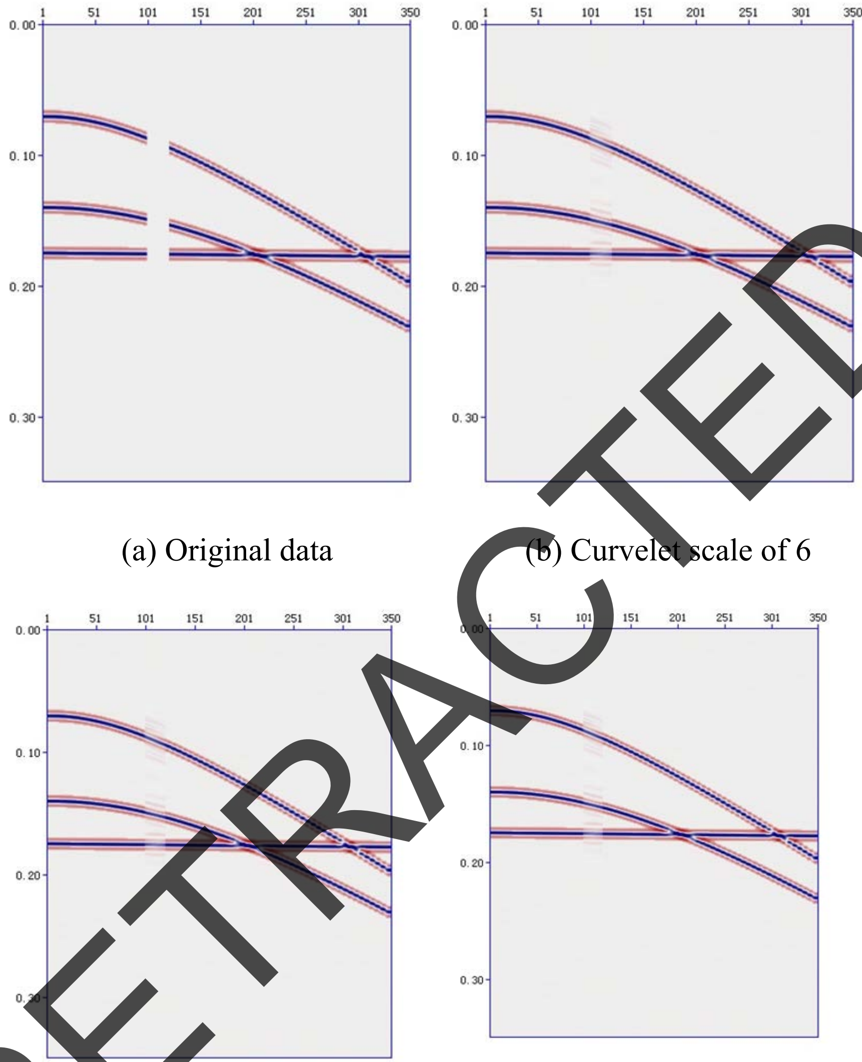


(h) Reconstruction results for 40 missing traces

Figure 12 Reconstruction results for seismic data with different numbers of missing traces

Influence of curvelet scale

Figure 13 shows the influence of curvelet scale on data restoration for 20 missing traces, which account for 5.7% of total traces.



(a) Original data (b) Curvelet scale of 6

(c) Curvelet scale of 4 (d) Curvelet scale of 2

Figure 13 Reconstruction results for seismic data with different curvelet scales

Curvelet scale has little impact on data restoration. An increase in curvelet scale is helpful to the restoration of details, which is unnecessary in this test. Hence, data restoration with different scales yielded similar results.

Originally incomplete data on both sides of missing traces in the time domain led to an inaccurate measurement matrix. The gaps in recovered data represent missing information in original data. If original data are complete without null values on both sides of missing traces, data restoration will yield good results. Hence, original data integrity on both sides of missing traces in the time domain is a prerequisite to data restoration with good results.

Different numbers of missing traces were tested. For a data size of 350×350, restoration results are good for the number below 20, acceptable for the number of 30, and unsatisfactory for the number of 40. This means that the scale of curvelet basis is restrictive. Restoration results are good for the curvelet scale of 6 and missing traces less than 6%.

Field Data

OBN data (as shown in Figure 14) with single-ended spread have 1378 shots and 350 channels per shot. Group interval is 50 m; record length is 8000 ms; sampling interval is 2 ms. For the convenience of analysis, the data of the first shot (Figure 15(a)) were processed first. In Figure 15(b) with missing traces plotted in red, the number of continuous missing traces is 18 at most and 1 at least. We used 6 curvelet scales, 8 angles at the largest scale, threshold of 10, and 100 iterations for data restoration. The results are shown in Figure 16(a). Figure 16(b) shows the restoration results of compressed sensing using Fourier transform for comparison.

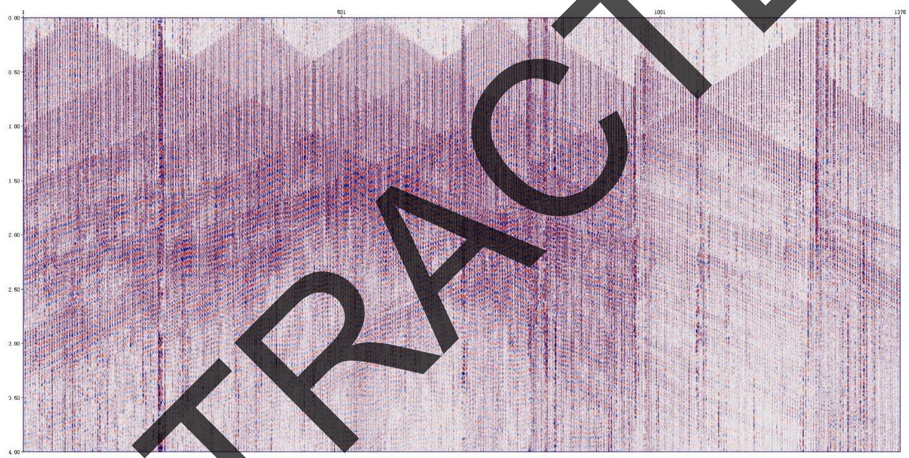
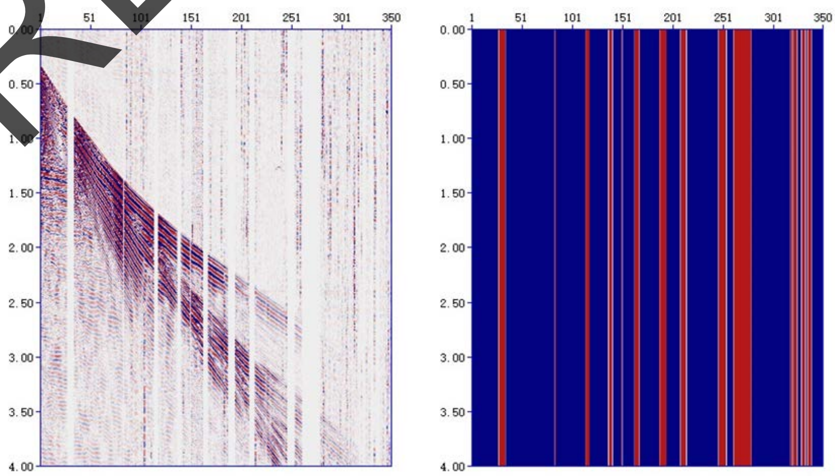
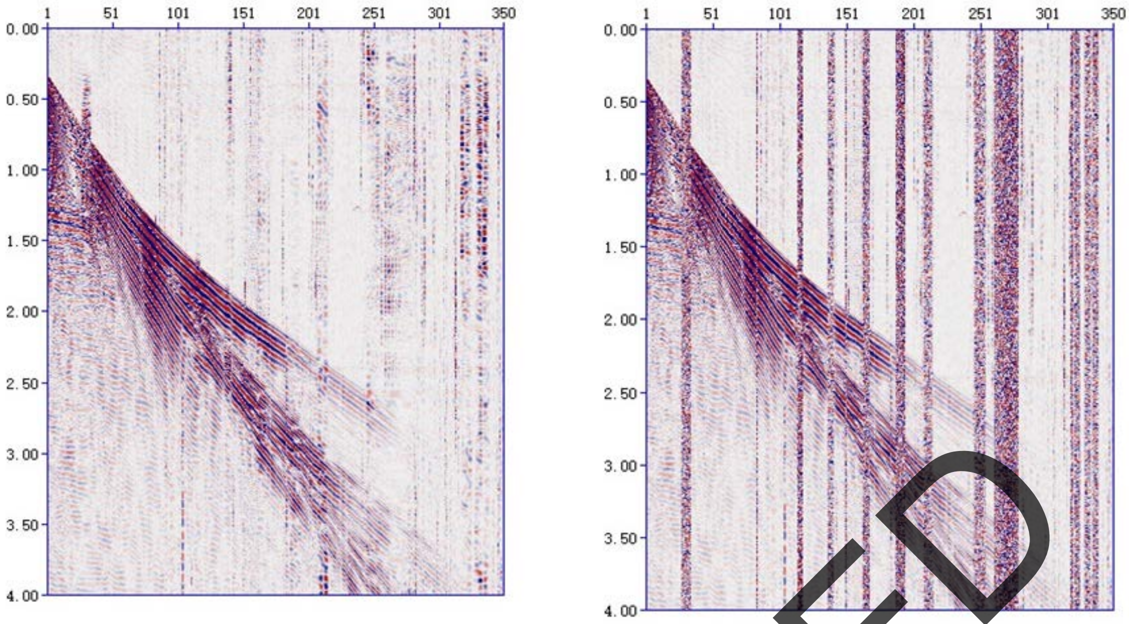


Figure 14 Field OBN data



(a) Single-shot data (b) Missing traces

Figure 15 Single-shot OBN data and missing traces



(a) Curvelet transform (b) Fourier transform

Figure 16 Reconstruction results of single-shot OBN data

As shown in Figure 16(a), OBN data missing is often attributed to node missing; hence, complete data on both sides of missing traces lead to good results of restoration. Except for one or two areas (e.g. the rightmost abnormal area in Figure 15(a)) with too many missing traces, missing traces in additional areas do not exceed 6% of total traces. Thus, restored data show relatively continuous events and consistent amplitude.

As shown in Figure 16(b), Fourier transform for data reconstruction yields more distinct aliases than curvelet transform. This is because according to the Nyquist sampling theorem, for the given dominant frequency and group interval, only when the number of missing traces is small can aliasing be avoided in data restoration based on Fourier transform. Owing to a large number of missing traces in field data, restored data in Figure 16(b) show serious aliases. This means that curvelet transform is more feasible for OBN data restoration based on compressed sensing.

Compressed sensing based on curvelet transform was performed to reconstruct OBN data of 1378 shots (as shown in Figure 17). Except for several areas with too many missing traces, restored data in additional areas show more continuous events and abundant information than original data (shown in Figure 14).

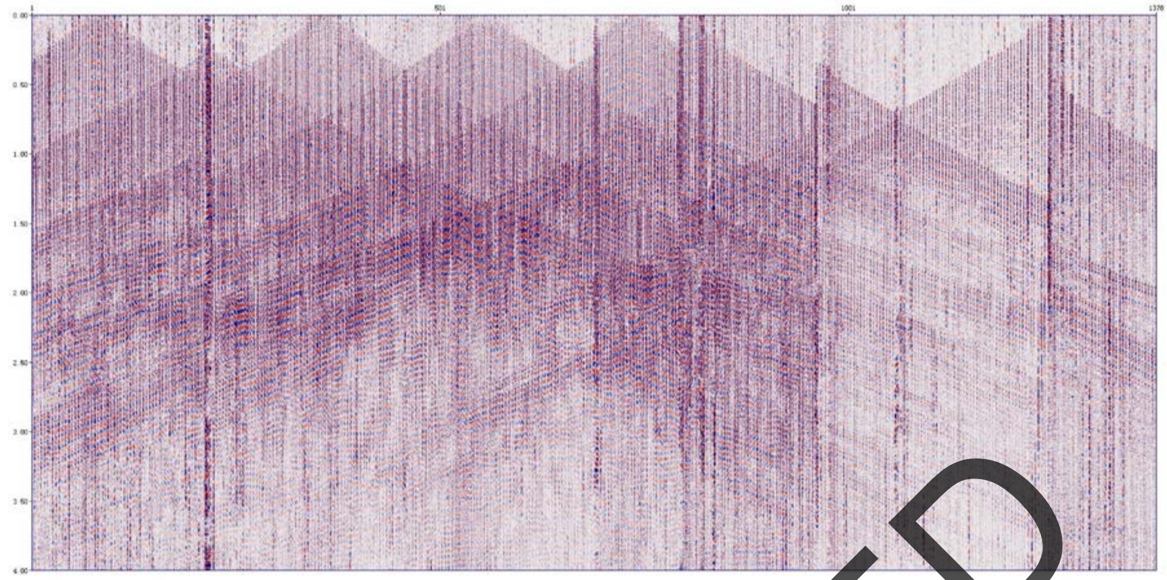


Figure 17 Reconstruction result of field OBN data

We established the sampling matrix of OBN data and used compressed sensing based on curvelet transform and iterative shrinkage-thresholding for OBN data restoration and reconstruction. As per model tests, data restoration is satisfactory in the context of complete data on both sides of missing traces and missing traces less than 6% of total traces. How to define threshold and curvelet scale depends on measured data. An application to OBN data of 1378 shots yielded good results of missing data restoration and reconstruction with more continuous events and abundant information than original data; compressed sensing was demonstrated to be effective for OBN data reconstruction.

CONCLUSIONS

To solve the problem of data missing in wireless OBN data acquisition, we developed a compressed sensing technique, which takes full advantage of the sparse representation of seismic data in the time-space transform domain, for field data restoration and reconstruction. This technique may replace the process of reshooting and re-acquisition to reduce the cost and improve the efficiency of seismic prospecting. Centering on the theory of compressed sensing, we discussed the basic theory of compressed sensing, methods related to three steps in compressed sensing, and application to model data and field data. Following conclusions are arrived.

(1) As per theoretical analysis, compressed sensing can reconstruct original data at the frequencies far below the Nyquist frequency, which cannot be accomplished using conventional methods. This is the greatest strength of compressed sensing; thus, this technique is significant to seismic data restoration and reconstruction.

(2) Compressed sensing mainly includes three steps: sparse representation of signals, measurement matrix design, and signal restoration and reconstruction, each of which can be implemented using many methods. As per comparative analysis, curvelet transform features multi-scale, locality, and multi-direction, and can accomplish optimum non-linear approximation of seismic wavefront; random sampling mitigates and even eliminates aliasing; iterative shrinkage-thresholding features simple operation, fast iteration, and good performance for noisy data. For OBN data with curved events and missing traces of random distribution, compressed sensing based on curvelet transform, random sampling, and iterative shrinkage-thresholding is the most feasible method for data restoration and reconstruction.

(3) We established the sampling matrix of OBN data and used compressed sensing based on curvelet transform and iterative shrinkage-thresholding for OBN data restoration and reconstruction. Model tests show satisfactory data restoration in the context of complete data on both sides of missing traces and missing traces less than 6% of total traces. How to define threshold and curvelet scale is dependent on measured data. An application to OBN data of 1378 shots yielded good results of missing data restoration and reconstruction. Compared with original data, improved event continuity and information content demonstrate the validity of compressed sensing for data restoration and reconstruction.

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