CORRECTION OF PRIMARY AMPLITUDES FOR PLANE-WAVE TRANSMISSION LOSS THROUGH AN ACOUSTIC OR ABSORPTIVE OVERBURDEN WITH THE INVERSE SCATTERING SERIES INTERNAL MULTIPLE ATTENUATION ALGORITHM: AN INITIAL STUDY AND 1D NUMERICAL EXAMPLES

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ABSTRACT

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The objective of extracting the spatial location of a reflector, and its local angle-dependent reflection coefficient, from seismic data, depends on the ability to identify and to remove the effect on primary amplitudes of propagation down to and back from the reflector. All conventional methods that seek to correct for such transmission loss require estimates of the properties of the overburden. In this paper we propose a fundamentally new approach that will in principle permit correction of primaries for such transmission loss without requiring overburden properties as input. The approach is based on the amplitude of the first term of the inverse scattering series internal multiple attenuation algorithm, which predicts the correct phase and approximate amplitude of first order internal multiples. The amplitude is estimated to within a factor determined by plane wave transmission loss down to and across the reflector producing the event's shallowest downward reflection. Hence, the amplitude difference between a given predicted and actual multiple, both of which are directly available from the data and the algorithm output, in principle contain all necessary information to correct specific primary reflections for their overburden transmission losses. We identify absorptive overburdens/media as requiring particular focus, so as a first step, previous amplitude analysis of the internal multiple attenuation algorithm is here extended to include stratified absorptive media. Using this newly derived relationship between predicted and actual internal multiples, and existing results for acoustic/elastic media, correction operators, to be applied to specific, isolated primaries in both types of media, are then computed using combinations of multiples and their respective predictions. We illustrate the approach on synthetic data for the absorptive case with three earth models with different Q profiles. Further research into the ampli-

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tudes of the plane wave internal multiple predictions in 2D and 3D media is a likely pre-requisite to field data application of this concept-level algorithm.

KEY WORDS: absorption, internal multiples, inverse scattering series, transmission losses.

INTRODUCTION

A primary is a recorded seismic event whose history (Fig. 1) can be roughly subdivided into: propagation down from the source through the overburden, reflection at a target, and propagation back through the overburden up to the receiver:

$$\text{Primary} = [\text{Transmission Down}] \times [\text{Reflection}] \times [\text{Transmission Up}] \; . \quad (1)$$

In exploration seismology primaries are the main source of subsurface information, and are used for structural mapping, parameter estimation, and, ultimately, petroleum delineation at the target. Techniques of *migration-inversion* (Weglein and Stolt, 1999) accomplish these goals by first generating maps of seismic reflectors at depth, typically positioning at these reflectors reflection coefficients as functions of angle, and, second, by using this behavior to determine local contrasts in medium properties. Therefore, an important part of migration-inversion is the processing of primary amplitudes, which are themselves essentially described by eq. (1), to remove the effects of transmission down to and back from the point of reflection, "laying bare" the reflection coefficient information so that it may be used in parameter estimation.

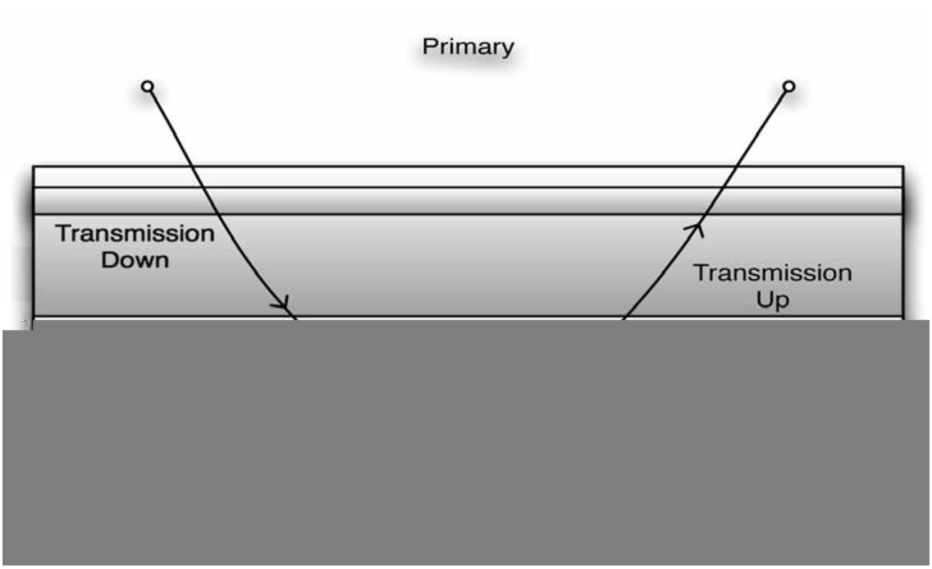


Fig. 1. Sketch of a primary. Amplitudes are determined by the material property contrast at the point of reflection, and propagation down and back through the overburden.

This removal as it is conventionally accomplished requires an accurate estimate of all medium properties above the target.

In this paper we describe an approach for the correction of primary amplitudes for transmission through various types of overburden, that avoids the requirement for prior characterization of overburden properties, thus aiding otherwise conventional migration-inversion methods. We seek a corrective operator, derivable directly from the data, of the form

$$\text{Corrective Operator} = ([\text{Transmission Down}] \times [\text{Transmission Up}])^{-1} \ , \quad (2)$$

which, when applied to a particular primary as modeled by eq. (1), provides the reflection coefficient information required by the inversion component of migration-inversion:

$$\text{Corrected Primary} = \text{Primary} \times \text{Corrective Operator} = [\text{Reflection}] \ . \quad (3)$$

Our approach derives from the inverse scattering series internal multiple attenuation algorithm (Araújo et al., 1994; Araújo, 1994; Weglein et al., 1997; 2003). Multiples are defined, herein, as events which have experienced at least one downward reflection. When at least one of these downward reflections takes place at the free surface, the event is a *free-surface* multiple*, otherwise the event is an *internal multiple*.

The *order* of a free-surface multiple refers to the number of downward reflections experienced by the event at the free surface, and the order of an internal multiple refers to the number of downward reflections experienced by the event anywhere in the subsurface (Weglein et al., 2003); e.g., first order internal multiples have one downward reflection, etc. The inverse scattering series has the ability to eliminate all multiples without a priori subsurface information (Weglein et al., 2003). The inverse series algorithm for free-surface multiples eliminates a single order of free-surface multiples with a single algorithm term (of the same order). In contrast, each order of internal multiples requires a series for its removal. For instance, the internal multiple attenuation algorithm is a series, whose first term predicts the correct time and approximate amplitude of all first order internal multiples, and prepares the higher order multiples for attenuation by higher order terms in the algorithm**.

* This definition is contingent on prior removal of ghosts.

** Research has additionally progressed towards an elimination algorithm. Ramirez and Weglein (2005a); Ramirez (2007) have provided a closed-form elimination algorithm for a subset of first-order internal multiples, which eliminates internal multiples generated at the shallowest reflector in the earth and improves the attenuation of internal multiples generated at deeper reflectors. Further aspects of the internal multiple attenuation algorithm have been reported in the literature by (Carvalho et al., 1991; Matson, 1997; Weglein et al., 1997; Weglein and Matson, 1998; Kaplan et al., 2005; Nita and Weglein, 2005; Weglein and Dragoset, 2005).

Our proposed primary correction approach derives from the properties of the first term of the internal multiple attenuation algorithm. The precise difference between the actual amplitude of an internal multiple and the amplitude predicted by the first term of the algorithm, for plane wave data in an acoustic medium (Weglein et al., 2003; Nita and Weglein, 2005), is a direct expression of plane wave transmission losses down to and across the reflector where the multiple's shallowest downward reflection has taken place (Weglein and Matson, 1998; Weglein et al., 2003; Ramirez and Weglein, 2005b;a). This means that the amplitude difference between a given multiple and its prediction, both of which are directly available from the data and the algorithm output, in principle contains all the information necessary to correct specific primary reflections for their overburden transmission losses*. The main goal of this paper is to use this information to construct a corrective operator essentially of the form described in eq. (2). In doing this early-stage research, we assume that wavelet estimation and deconvolution, instrument response analysis, and de-ghosting have already been carried out, and that the requisite data events have been identified and can be separately studied.

There are additional potential benefits associated with this idea: first, the information is a byproduct of an existing part of the wave-theoretic processing flow (the de-multiple phase) and comes at no additional cost. Second, this information becomes available at a convenient point during processing, just prior to its likely use in primary processing/inversion. Third, it is consistent with wave-theoretic processing. Fourth, it is not restricted to a production setting, but is also applicable in reconnaissance and exploration settings.

We have in particular found that the design of the operator depends on whether or not the overburden is absorptive. Hence, after briefly re-stating the properties of the internal multiple attenuation algorithm as studied for acoustic/elastic media (Araújo et al., 1994; Araújo, 1994; Weglein et al., 1997; 2003; Ramirez and Weglein, 2005b;a), we begin with the important preliminary step of deriving, for the first time, expressions for the difference between the internal multiple attenuation algorithm prediction and the actual amplitude of the multiple event for a layered anelastic medium. This allows operators appropriate for either medium type to be derived.

We next work with the formulas expressing the difference between actual and predicted multiples, demonstrating that, when combined recursively, they may be used to produce correction operators ready for multiplicative application (in the frequency domain) to specific primaries. The operators are built only from the data and the output of the internal multiple algorithm (which itself has

* The use of the discrepancy for correcting for overburden effects was first suggested by Dennis Corrigan following discussions on the analytic example presented by A. Weglein at CWP and ARCO, later published by Weglein and Matson (1998).

required no overburden information). Different operators are made for the acoustic/elastic vs. absorptive cases. Following this, we illustrate the procedure on synthetic data for the absorptive case, examining the form and effect of the correction operators. We conclude with remarks on a path forward for making this potentially powerful approach practical.

AMPLITUDES PREDICTED BY THE MULTIPLE ATTENUATION ALGORITHM

The first term in the internal multiple attenuation algorithm acts non-linearly on reflection seismic data to calculate the exact phase and approximate amplitude of all orders of internal multiples:

$$\begin{aligned} b_{3IM}(k_g,k_s,q_g+q_s) = {} & [1/(2\pi)^2] \int_{-\infty}^{\infty} dk_1 e^{-iq_1(z_g-z_s)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(z_g-z_s)} \\ & \times \Big[\int_{-\infty}^{\infty} dz_1' b_1(k_g,k_1,z_1') e^{i(q_g+q_1)z_1'} \\ & \times \int_{-\infty}^{z_1'-\epsilon} dz_2' b_1(k_1,k_2,z_2') e^{-i(q_1+q_2)z_2'} \\ & \times \int_{z_2'+\epsilon}^{\infty} dz_3' b_1(k_2,k_s,z_3') e^{i(q_2+q_s)z_3'} \Big] \ , \end{aligned} \tag{4}$$

where $q_g = \text{sgn}(\omega)\sqrt{[(\omega/c_0)^2 - (k_g)^2]}, q_s = \text{sgn}(\omega)\sqrt{[(\omega/c_0)^2 - (k_s)^2]}, q_s$, k_g and k_s are the horizontal wavenumbers conjugate to receiver and source coordinates (x_g,x_s), respectively, and ϵ is a small positive quantity. The input for the internal multiple attenuation algorithm is b_1, which is created from the pre-stack reflection seismic data. It is constructed as follows: the surface recorded data, deghosted and without free surface multiples, $D(x_g,x_s,t)$, is Fourier transformed over all variables, to produce $D(k_g,k_s,\omega)$. A change of variables is made, to $D(k_g,k_s,q_g + q_s)$, after which b_1 is defined as $b_1(k_g,k_s,q_g + q_s) = D(k_g,k_s,q_g + q_s)(2iq_s)$; b_1 is then inverse Fourier transformed over $q_g + q_s$ to pseudo-depth. The result, $b_1(k_g,k_s,z)$, is used as input in eq. (4), and the output, b_{3IM}, is the predicted internal multiple data set, produced without knowledge of earth material properties or structure and it accommodating all earth model types that satisfy the convolutional model (Ramirez and Weglein, 2005b).

The relationship between the predicted and the actual multiple amplitude

Being the first term in a series that removes first order internal multiples without subsurface information, the internal multiple attenuation algorithm provides the capability to predict the exact time of all first order internal multiples and it is the first term to predict the amplitudes of the first order internal multiples. Weglein and Matson (1998) and Ramirez and Weglein (2005b) examined the difference between the actual amplitudes of internal multiples and those of the internal multiple attenuation algorithm predictions. The latter authors called the difference the amplitude factor, and showed that it is related to the transmission coefficients down to and across the multiple generator interface (Weglein and Matson, 1998; Ramirez and Weglein, 2005b). The difference can be understood intuitively by considering the way the algorithm builds its prediction. Consider Fig. 2. On the left panel we sketch an internal multiple and the three primaries that are used in the algorithm to predict it. The generator is interface 2. The multiple has the path abcdijkl. The algorithm predicts the multiple by multiplying the amplitudes of the three primaries, adding the phases of the deeper two, abcdef and ghijkl, and subtracting the phase of the shallower, ghef. The phase of the actual multiple and the predicted multiple are therefore identical. However, the amplitude of the actual multiple,

$$T_{ab}T_{bc}R_{cd}(-R_{he})R_{ij}T_{jk}T_{kl} \ ,$$

and the multiplied amplitudes of the primaries in the prediction,

$$[T_{ab}T_{bc}R_{cd}T_{de}T_{ef}] \times [T_{gh}R_{he}T_{ef}] \times [T_{gh}T_{hi}R_{ij}T_{jk}T_{kl}],$$

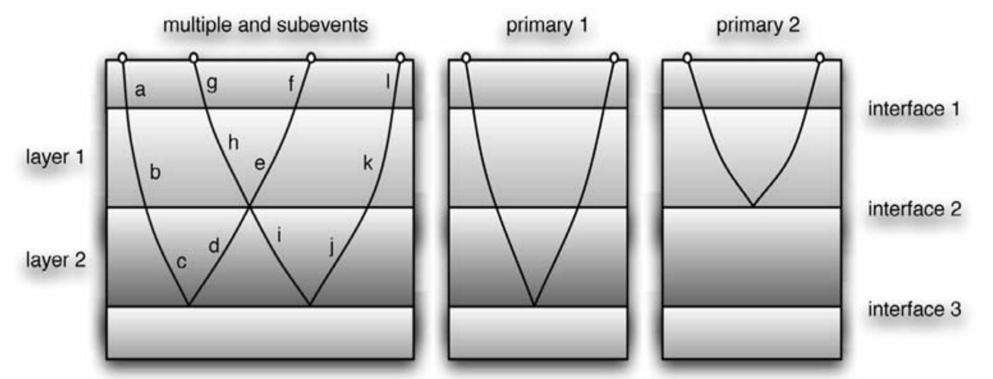
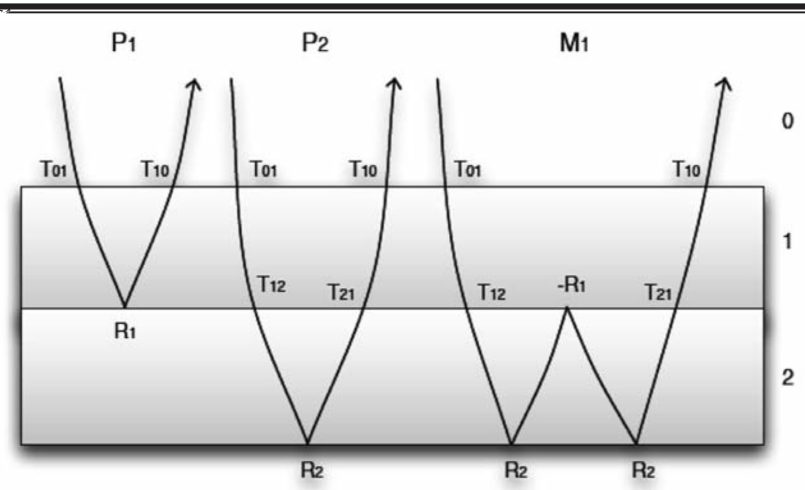


Fig. 2. Schematic diagram of primaries and internal multiples in a stratified medium. Left panel: an internal multiple and the primary subevents used to predict it. Middle and right panels: associated primaries whose amplitudes may be corrected using the discrepancy between the amplitudes of the predicted and actual multiple on the right.

clearly differ in that the actual multiple does not experience the transmission history of the shallower primary. That is, the terms T_{de}, T_{ef}, T_{gh}, T_{hi} in the prediction are extraneous. We note that this includes transmission across the generating interface.

Let us next depart from schematics and consider the general accounting of this behavior for predicted multiples within an arbitrary stack of layers provided by Ramirez and Weglein (2005b). Fig. 3 shows a 1D acoustic model



The index 2 anticipates our later use of this factor for corrective purposes, and signifies that the second interface is the generator. With this terminology the amplitude factor expressing the discrepancy between the predicted and actual amplitudes of an internal multiple generated at the j-th interface in a stack of layers is

$$AF_j = \begin{cases} T_{01}T_{10} & \text{for } j = 1; \\ \Pi_{i=1}^{j-1}(T_{i,i-1}^2 T_{i-1,i}^2)T_{j,j-1}T_{j-1,j} & \text{for } 1 < j < J \ , \end{cases} \qquad (8)$$

where J is the total number of interfaces in the model. Presently we will manipulate this factor to become a correction operator for primary amplitudes.

Extension of the amplitude analysis to absorptive media

Ramirez and Weglein (2005b) assume an acoustic medium, in which plane-wave transmission losses are local, occurring at the point at which the wave crosses a contrast in material properties. For an absorptive stack of layers, in which transmission loss occurs over the entire course of propagation, an extension of their results is required. In later sections we will see that this minor theoretical alteration leads to an important practical difference when the predicted-actual amplitude discrepancy is exploited.

In order to study the transmission coefficients in an anelastic medium we select an intrinsic attenuation model to describe amplitude and phase alterations in a wave due to friction. These alterations are modeled by a generalization of the wavefield phase velocity to a complex, frequency-dependent quantity parameterized in terms of Q. A reasonably well-accepted Q model (Aki and Richards, 2002) alters the scalar propagation constant of the j-th layer, $k_j = \omega/c_j(z)$, to

$$k_j = [\omega/c_j(z)][1 + F(\omega)/Q_j(z)] \ , \qquad (9)$$

where $F(\omega) = i/2 - (1/\pi)/\log(\omega/\omega_0)$. The reference frequency ω_0 may be considered a parameter to be estimated, or assumed to be the largest frequency available to a given experiment. The model divides propagation into three parts: a propagation component, an attenuation component, and a dispersion component.

With this extended definition of k_j, and assuming that in Fig. 3 the two bottom layers are anelastic, we again construct the prediction. It is convenient to re-define the transmission coefficient of a given interface to incorporate absorptive amplitude loss within the layer *above* that interface. For instance, the coefficients T_{12} and T_{21} of the previous section derived using the k_j of eq. (9), become:

$$\mathcal{T}_{12} = \left[\frac{2c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1}}{c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1}+c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1}}\right] \overbrace{e^{-\frac{\omega}{2Q_1c_1}(z_1-z_0)}}^{\text{attenuation component}} \; e^{\frac{i\omega}{\pi Q_1 c_1}\log(\frac{\omega}{\omega_0})(z_1-z_0)} \tag{10}$$

$$\mathcal{T}_{21} = \left[\frac{2c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1}}{c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1}+c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1}}\right] \underbrace{e^{-\frac{\omega}{2Q_1c_1}(z_1-z_0)}}_{\text{attenuation component}} \; e^{\frac{i\omega}{\pi Q_1 c_1}\log(\frac{\omega}{\omega_0})(z_1-z_0)} . \tag{11}$$

We make particular note of the amplitude dependence (via the attenuation component) of this definition of transmission coefficients on the thickness of the layer *overlying* the interface in question. With this extension, we have essentially the same amplitude factor, for instance AF_2, in the anelastic case as we did in the elastic case. By analogy with eq. (7):

$$AF_2 = [T_{01}T_{10}]^2 \, \mathcal{T}_{12} \, \mathcal{T}_{21} \quad . \tag{12}$$

Provided that this re-definition of the absorptive transmission coefficients is adopted, the amplitude factors and internal multiple attenuation error analysis for the general absorptive stack of layers at normal incidence is given again by eq. (8).

CORRECTION OF PRIMARY AMPLITUDES USING INTERNAL MULTIPLES

Let us make two comments about the amplitude error analysis above. First, we see that the discrepancy between the predicted and the actual multiple for a given generator is directly related to the transmission losses experienced by a primary associated with that generator. Second, we note that the discrepancy, characterized by the amplitude factor AF, is available directly from the data and the output of the internal multiple attenuation algorithm. In this section we use the information in the various AF factors as a direct means to correct the amplitude of the primary associated with the generator for transmission effects, in the sense we have put forward in the introduction.

We define what will become the primary correction operator, PCO, to be built recursively from the data-determined AFs:

$$PCO_n = 1/(AF_n \times PCO_{n-1}) \quad , \tag{13}$$

with the terminating definition:

$$PCO_0 = 1 \quad . \tag{14}$$

Expanding this operator over several orders n clarifies that it will indeed act as a correction operator when applied to a primary whose upward reflection has occurred near the n-th interface.

We find that the precise primary which should be corrected with the n-th operator depends on whether the medium is assumed to be absorptive or not. We next treat these cases in turn. It is useful to index primaries from 0 upward. In the scheme in Fig. 2, the 0-th primary reflects upward at interface 1.

Correction of primaries in acoustic/elastic media

Consider once again the multiple sketched in Fig. 2, whose generator is interface 2. Setting n = 2, expanding eq. (13), and employing the alphabetical indices we use in the figure, we have

$$PCO_2 = 1/T_{gh}T_{hi}T_{de}T_{ef} \ . \tag{15}$$

If the medium is acoustic/elastic, we note that for the primary depicted in the middle panel of Fig. 2, the "last" overburden effect on the event before the reflection at interface 3 is the transmission through interface 2, and the "first" overburden effect on the event after the reflection is again transmission through interface 2. Consequently, PCO_2 is exactly appropriate as an operator to correct this (middle panel of Fig. 2) primary. More generally, in the acoustic/elastic case, the operator PCO_n in eq. (13) corrects the n-th primary, leaving the n-th reflection coefficient "bare" and suitable as input to other inverse procedures:

$$Rn = PCO_n \times P_n \ . \tag{16}$$

Correction of primaries in absorptive media

Next, let us suppose that the medium in Fig. 2 is absorptive, and again consider PCO_2. Recall that we may maintain the same form for the amplitude discrepancy between predicted and actual multiples in absorptive media and thereby this operator, PCO_2, provided we alter the transmission coefficients of a given interface to include absorptive propagation through the layer above that interface.

With this arrangement PCO_2 is evidently no longer appropriate as an operator to correct primary 2, i.e., the primary depicted in the middle panel of Fig. 2, because it does not account for absorptive propagation through the layer between the reflection and the multiple generator.

To maintain the usefulness of the operator, we instead make an approximation. We assume that in an absorptive medium, the effect of the local transmission coefficient at a boundary on the amplitude of a primary is dwarfed by the effect of absorptive propagation. With that assumption we may simply change the primary being corrected by PCO_2 to the one depicted in the right panel of Fig. 2. This statement is true to within the combined local transmission coefficient down and up across interface 2. More generally, in the absorptive case, the (now frequency-dependent) operator PCO_n in eq. (13) corrects the $n-1$-th primary:

$$R_{n-1}(\omega) = PCO_n(\omega) \times P_{n-1}(\omega) \quad . \tag{17}$$

SYNTHETIC EXAMPLES

In this section, we illustrate with simple synthetic examples the steps necessary to correct a primary for absorptive transmission losses, using a multiple and the internal multiple attenuation algorithm prediction. We generate zero-offset traces from plane waves normally incident on three layered models with the geometry of the model in Fig. 1, assuming the waves behave in accordance with the propagation constant in eq. (9), and using the layer parameter values in Table 1. We include the two primaries and the first order internal multiple. The traces are wavelet deconvolved, and bandlimited (3-50 Hz). Fig. 4 shows the traces generated for each model, which differ in their Q values, ranging from relatively low attenuation to relatively high attenuation. The arrival times of the two primaries and the multiple are approximately 1.5s, 2.3 s and 2.9 s, respectively.

Table 1. Absorptive earth models.

Depth (m)	c (m/s)	Q1	Q2	Q3
000 - 500	1500	∞	∞	∞
500 - 1422	2200	200	100	50
1422 - 2422	2800	100	50	25
2422 - ∞	3300	50	25	10

With the knowledge that the medium is absorptive, and in accordance with our arguments in the previous section, we use the predicted multiples to correct the amplitude of the shallower primary. The prescription is:

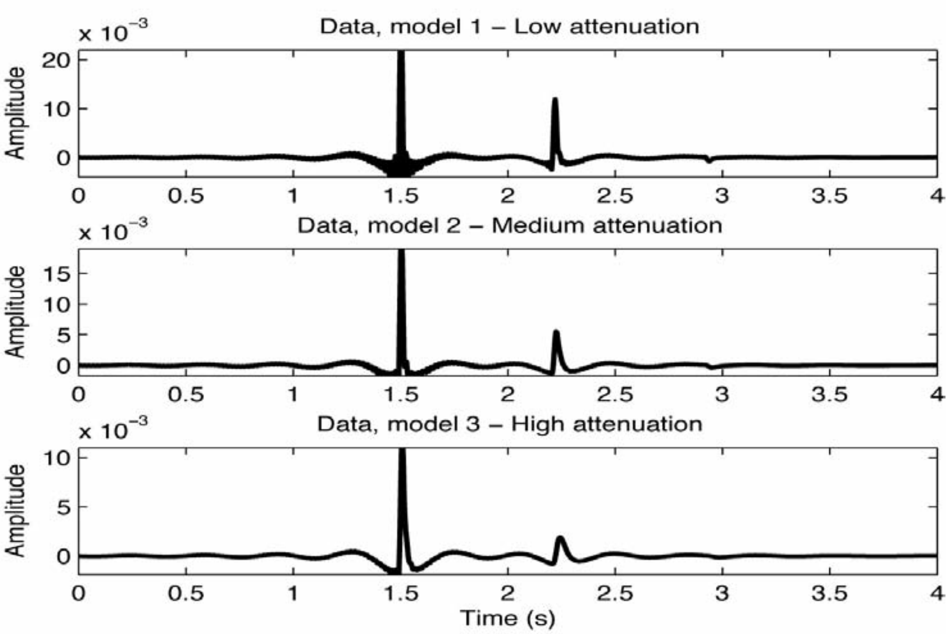


Fig. 4. Synthetic data: two primaries and one multiple are generated for each of three models.

1. Each trace is used as input to the internal multiple attenuation algorithm, generating predictions of the internal multiples.

2. Each internal multiple and its prediction are isolated and their spectra calculated.

3. The reciprocal of the ratio between the spectra of each internal multiple and its prediction is taken. By eq. (13), this is the appropriate correction operator PCO.

4. The shallower primary is isolated, and the operator is applied to its spectrum.

We compare the result to an equivalent primary which we model in the absence of all effects of transmission through the overburden. Fig. 5 illustrates the uncorrected, shallower primary from each of the three models. We predict the multiple with the attenuation algorithm, and isolate both this prediction and the original multiple from the trace, and compute their spectra (Fig. 6). The prediction evidently contains a greater level of attenuation then the actual multiple. This is in agreement with the extra transmission paths involved in the

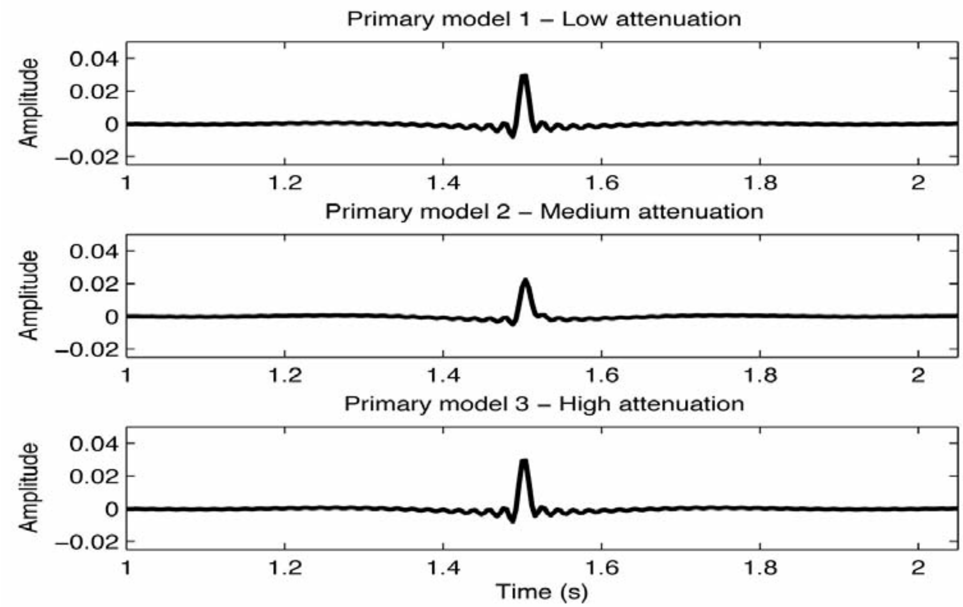


Fig. 5. Primary generated at interface 1 in Fig. 3 for all models. These are the events we intend to compensate for transmission losses using the discrepancy between the actual multiple generated at interface 1 and the prediction of the internal multiple attenuation algorithm.

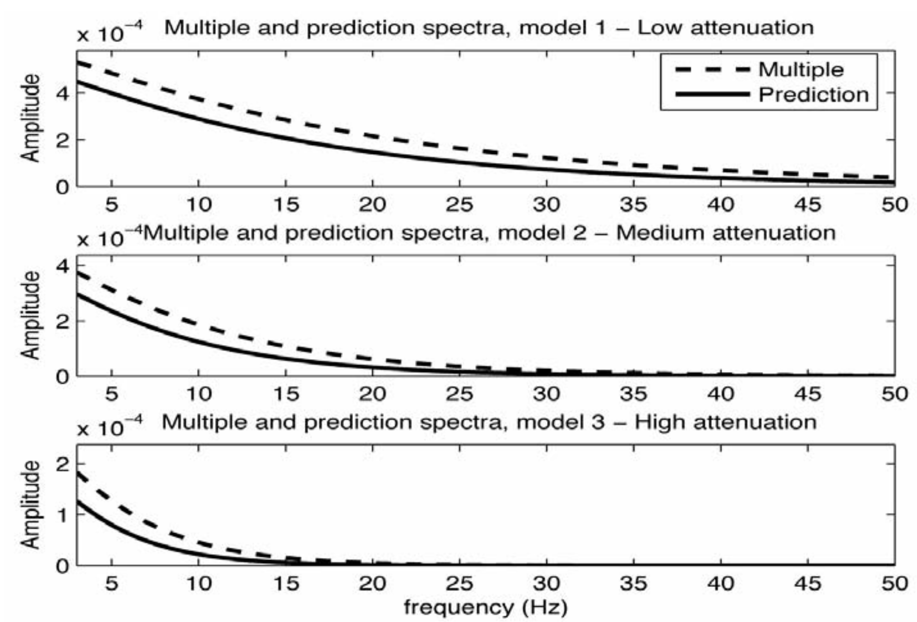


Fig. 6. The spectra of the multiple and its prediction from the internal multiple attenuation algorithm algorithm. The ratio between each pair of curves will be used for creating an operator for correcting the primaries of its transmission losses.

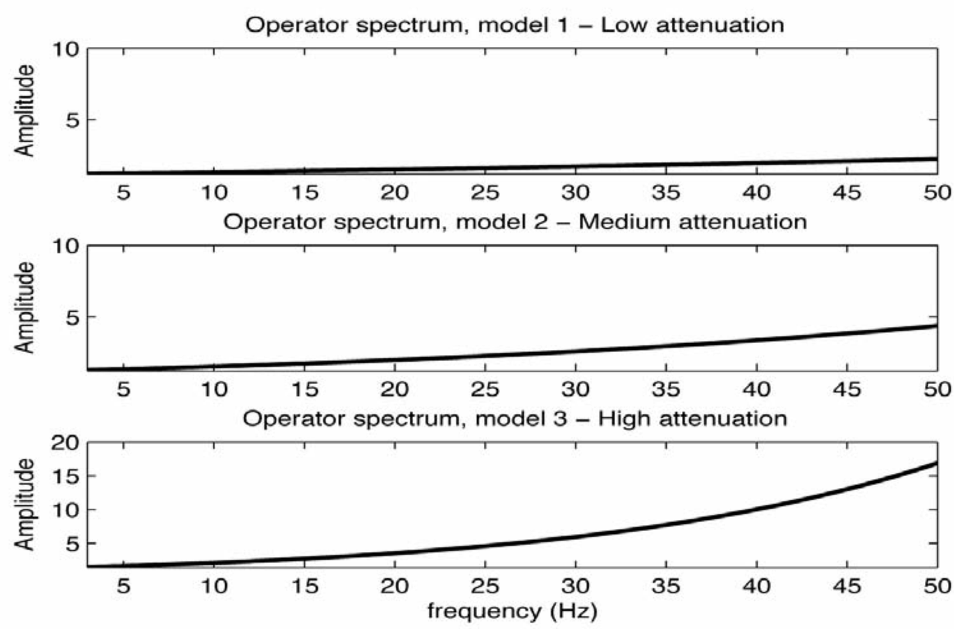


Fig. 7. Primary correction operators, generated from the ratio of the spectrum of the actual multiple and its prediction, as in Fig. 6.

prediction, as discussed above. The frequency dependence of this discrepancy will form the basis for the correction of the shallower primary, which, indeed, will have a distinct Q-compensation flavor. Fig. 7 illustrates the spectra of the primary correction operators derived from these quantities, and Fig. 8 illustrates the spectra of the shallower primaries for each model, before and after the correction. The recovery of high frequencies is notable. In Fig. 9, we illustrate the corrected primaries after inverse Fourier transforming to the time domain, and compare the results against their idealized counterparts constructed without transmission losses. Figs. 10 to 12 illustrate in close succession the original primary in the data (top panel), the corrected primary (middle panel) and the idealized primary (bottom panel), for all models. We point out that the discrepancy between the corrected primaries and idealized primaries is of a form and magnitude expected given our absorptive correction approximation, which neglects the local transmission through the boundary nearest the primary's point of reflection.

CONCLUSIONS

In this paper we have presented a procedure for correcting a primary for transmission losses using internal multiples and the output of the inverse scattering series internal multiple attenuation algorithm.

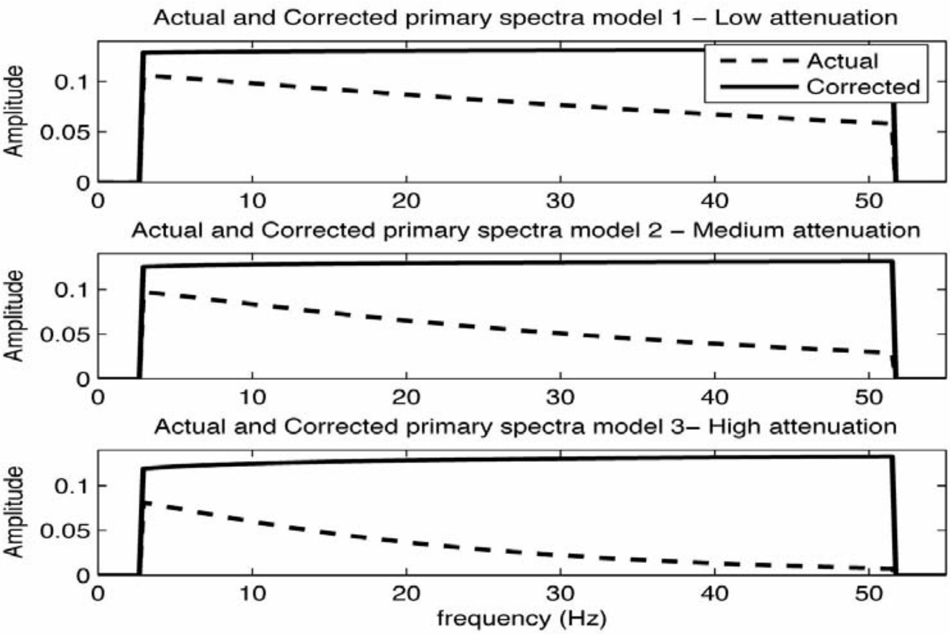


Fig. 8. Spectra of the original primaries for each model, and the primaries corrected by the operators depicted in Fig. 7.

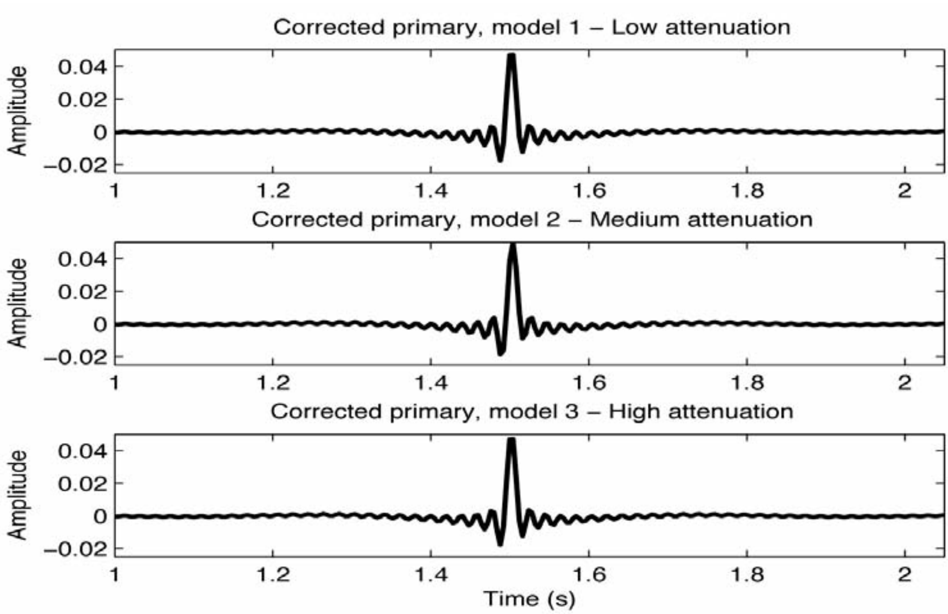


Fig. 9. The corrected primaries after the application of the operators in Fig. 7.

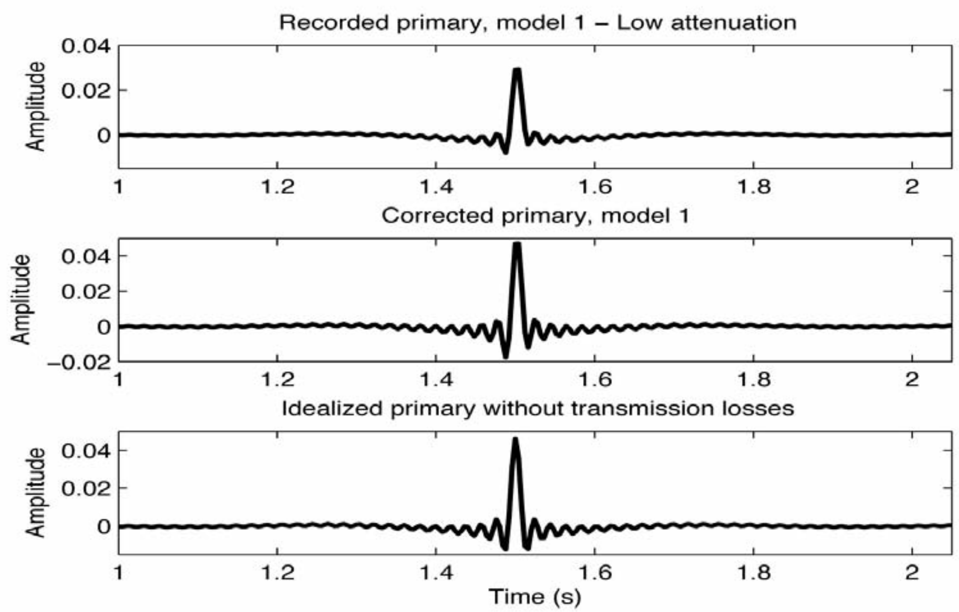


Fig. 10. A comparison of the actual primary (top curve), the corrected primary (middle curve) and the idealized primary, generated without transmission i(bottom curve) for model 1, the low attenuation case.

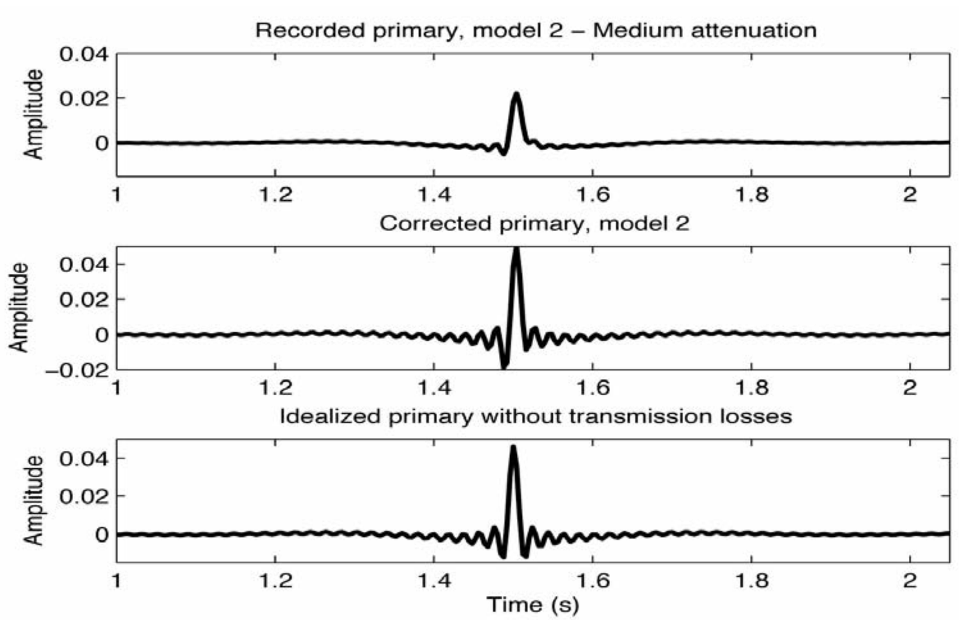


Fig. 11. A comparison of the actual primary (top curve), the corrected primary (middle curve) and the idealized primary, generated without transmission i(bottom curve) for model 2, the medium attenuation case.

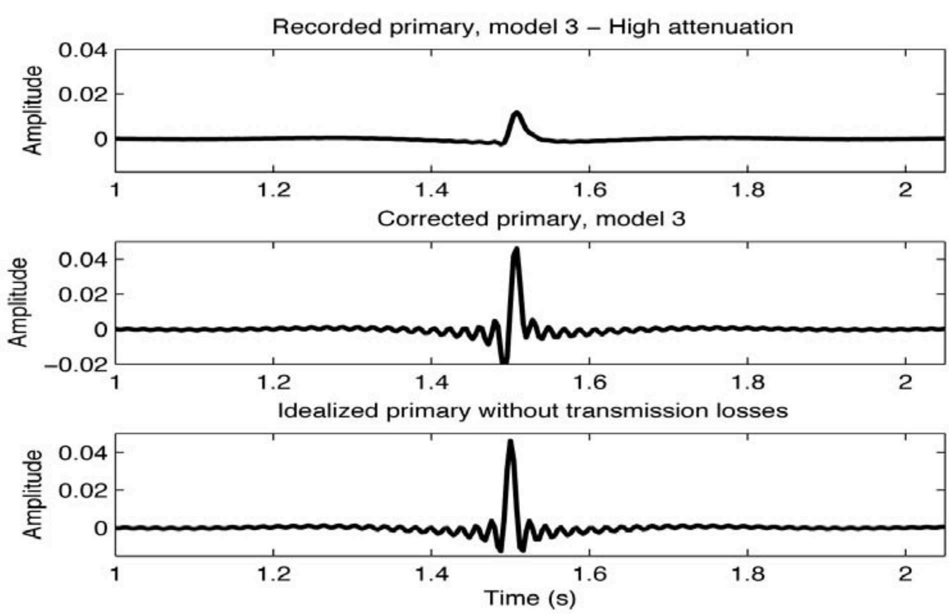


Fig. 12. A comparison of the actual primary (top curve), the corrected primary (middle curve) and the idealized primary, generated without transmission i(bottom curve) for model 3, the high attenuation case.

We have made particular mention and use of the distinction between situations involving significant absorption and situations that are largely acoustic or elastic. In spite of this broad categorization (that we have found to be practically important), one of the strengths of the approach is that it will act to correct transmission losses whatever their physical origin or mechanism, without requiring a precise model. In this sense the approach is truly data-driven - the events in the data, in comparison to one another, "decide" what the transmission loss must be.

Our simple numerical results are encouraging and motivate examination of the approach in the presence of more complex media, both absorptive and otherwise. The main tool in this approach, the internal multiple algorithm, is immediately applicable in multiple dimensions, and since the amplitude error is in terms of plane wave transmission coefficients, a plane wave decomposition of 2D and/or 3D data will likely suffice to extend the method. Nevertheless, detailed extension of the approach stands as ongoing and future research. For these reasons in particular, we identify field data testing as a medium-term to long-term goal, contingent on the fundamental study of the internal multiple attenuation amplitudes in multiple dimensions.

REFERENCES

Aki, K. and Richards, P.G., 2002. Quantitative Seismology, 2nd ed. University Science Books, San Francisco.

Araújo, F.V., 1994. Linear and non-linear methods derived from scattering theory: backscattered tomography and internal multiple attenuation. Ph.D. thesis, Universidade Federal da Bahia, Brazil.

Araújo, F.V., Weglein, A.B., Carvalho, P.M. and Stolt, R.H., 1994. Internal multiple attenuation. Extended Abstr., 56th AEGE Conf., Vienna: H036.

Carvalho, P.M., Weglein, A.B. and Stolt, R.H., 1991. Examples of a nonlinear inversion method based on the T-matrix of scattering theory: Application to multiple suppression. Expanded Abstr., 61st Ann. Internat. SEG Mtg., Houston: 1319-1322.

Kaplan, S.T., Robinson, W. and Innanen, K.A., 2005. Optimizing internal multiple attenuation algorithms for large distributed computing systems. Mission-Oriented Seismic Research Program (M-OSRP) Ann. Report.

Matson, K.H., 1997. An Inverse-scattering Series Method for Attenuating Elastic Multiples from Multi-component Land and Ocean Bottom Seismic Data. Ph.D. thesis, Univ. of British Columbia, Vancouver.

Nita, B.G. and Weglein, A.B., 2005. Inverse scattering internal multiple attenuation algorithm in complex multi-d media. Technical report, Mission-Oriented Seismic Research Project, Univ. of Houston.

Ramirez, A.C., 2007. I -Inverse scattering subseries for 1:Removal of internal multiples and 2:Depth imaging primaries; II -Green's theorem as the foundation of interferomerty and guiding new practical methods and applications. Ph.D. thesis, Univ. of Houston.

Ramirez, A.C. and Weglein, A.B., 2005a. An inverse scattering internal multiple elimination method: Beyond attenuation, a new algorithm and initial tests. Expanded Abstr., 75th Ann. Internat. SEG Mtg;, Houston: 2115-2118.

Ramirez, A.C. and Weglein, A.B., 2005b. Progressing the analysis of the phase and amplitude prediction properties of the inverse scattering internal multiple attenuation algorithm. J. Seismic Explor., 13: 283-301.

Weglein, A.B. and Matson, K.H., 1998. Inverse Scattering Internal Multiple Attenuation: An Analytic Example and Subevent interpretation. Abstr., SPIE Conf. Mathemat. Meth. Geophys. Imaging: 108-117.

Weglein, A.B. and Stolt, R.H., 1999. Migration-Inversion Revisited. The Leading Edge, 18: 950.

Weglein, A.B. and Dragoset, W.H., 2005. Multiple Attenuation. Geophysics reprint series. SEG, Tulsa, OK.

Weglein, A.B., Araújo, F.V., Carvalho, P.M., Stolt, R.H., Matson, K.H., Coats, R.T., Corrigan, D., Foster, D.J., Shaw, S.A. and Zhang, H., 2003. Inverse scattering series and seismic exploration. Inverse Problems, 19: R27-R83.

Weglein, A.B., Araújo Gasparotto, F., Carvalho, P.M. and Stolt, R.H., 1997. An inverse-scattering series method for attenuating multiples in seismic reflection data. Geophysics, 62: 1975-1989.