PSEUDO-ACOUSTIC WAVE EQUATION FOR MODELING AND REVERSE TIME MIGRATION IN TTI MEDIA

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ABSTRACT

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In recent years there has been a growing interest in the use of wave equations with anisotropy in the imaging of seismic data, due to the need to improve exploration and seismic data processing. Laboratory studies have indicated with convincing evidence that thin layers of shale introduces a polar anisotropy in the medium, which depends on the inclination of the layers with respect to an axis of symmetry. If the effect of anisotropy is not taken into account in the imaging procedure, the migrated section will present mispositioned reflectors.

To incorporate the effects of anisotropy in the seismic imaging, many migration algorithms based on the ray theory and on the solution of the wave equation, have been adapted from the isotropic case. Therefore, conventional methods of migration, including the reverse time migration, are prone to errors with some kinds of anisotropy in the medium, thus producing low resolution images and seismic mispositioned reflectors. Consequently, to produce images used to delineate reservoirs, for example, methods of migration that take into account the anisotropy of the medium must be implemented.

In this work we derive P-wave equations for TTI media starting from the exact dispersion equation for TTI media proposed by Tsvankin (1996). These new dispersion equations are valid for $\delta > \varepsilon$ (Thomsen's parameters) and strong anisotropy. Using the new equations for pure P-wave for TTI media, we migrated the BP-TTI synthetic data set with RTM technique using the rapid expansion method (REM). It significantly improved the migrations sections when compared with migrations that did not take into account the anisotropy of the medium.

KEY WORDS: anisotropy, TTI media, RTM, REM.

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INTRODUCTION

Acoustic pre-stack depth migration of PP seismic reflection is the best approach when a strong lateral variation of velocity and geological complexity are present. However, when the anisotropic character of the medium where the waves propagate can not be disregarded, an acoustic approach that yields to an accurate description of P-wave travel times is needed in order to avoid the solution of the expensive elastic wave equations.

Type of rocks, lateral and vertical facies variation or preferential direction of fracturing, are some of the causes that produce wave velocity dependence on the angle of propagation in the medium.

Although the vertical transversely isotropic (VTI) is the most simple media, in complex geology where the strata are overthrust and faults with steep dips and bodies with overhanding features are present, the assumption of a tilted transversely isotropy (TTI) media is more adequate because it takes into account the local symmetry of the model instead of just the global. More complex types of anisotropy are actually a challenge due to the difficulty to estimate the anisotropic parameters (Tsvankin et al., 2010).

The acoustic wave equation for VTI media, introduced by Alkhalifah (1998), can generate unwanted wave events. These events were initially categorized as numerical artefacts and, to avoid them, Alkhalifah (2000) proposed the introduction of a first isotropic layer in the model. Grechka et al. (2004) also demonstrated that the assumption of does not produce the complete elimination of the SV wave. To avoid the undesired SV wave component, different approximations of the expression developed by Tsvankin (1996) have been proposed to convert the fourth order wave equation into two separate second order equations: P- and SV-wave equations (Du et al., 2008; Liu et al., 2009; Du et al., 2010; Pestana et al., 2012).

In this work we present two approximations to the Alkhalifah (1998) equation for TTI media. The procedure used in the approximations reduces the fourth order differential equation used to model the P-wave to second order equations. This approach does not require the introduction of any auxiliary function because only one pair of roots of the fourth order dispersion relation is considered.

Comparisons of the velocity approximation of the P-wave to the exact Tsvankin (1996) expression and other approximations are presented. Modeling of the impulsive response shows that the proposed equations remove the diamond artifact and make the equation stable for any relation of the anisotropic parameters.

Numerical examples of seismic modeling and migration of TTI media are used to validate these new equations. The time extrapolation to numerically modeling and RTM for the P-wave was based on the rapid expansion method (REM) developed by Pestana and Stoffa (2010).

THEORY

When the wave velocity propagation depends on the angle between the wave vector and the vertical anisotropy symmetry axis, the medium is called VTI. A medium with anisotropic axis of symmetry rotated by an angle of ϕ, respective to the vertical axis, the medium is defined TTI.

In order to obtain the P- and SV-wave velocities, which depend on the phase angle and on the axis rotation angle ϕ, the expression for phase velocity in a VTI medium (Tsvankin, 1996) can be shifted by the angle ϕ. This expression is given by:

$$v^2(\theta,\phi)/v_{P0}^2 = 1 + \varepsilon\sin^2(\theta - \phi) - f/2$$

$$\pm (f/2)\{[1 + 2\varepsilon\sin^2(\theta - \phi)/f]^2 - [2(\varepsilon - \delta)\sin^2 2(\theta - \phi)/f]\}^{1/2} \quad , \tag{1}$$

where ε and δ are the Thomsen's parameters (Thomsen, 1986), v_{P0} and v_{S0} are the P- and SV-velocities at zero angle, and $f = 1 - v_{S0}^2/v_{P0}^2$. In eq. (1), the plus and minus signs correspond to P- and SV-wave velocities, respectively. Zhang et al. (2005) combined the P-wave velocity in (1) with the expressions for plane wave propagation

$$\sin^2\theta = v^2(\theta,\phi)k_x^2/\omega^2 \quad , \tag{2}$$

$$\cos^2\theta = v^2(\theta,\phi)k_z^2/\omega^2 \quad , \tag{3}$$

and assuming the acoustic approximation $f = 1$ (Alkhalifah, 1998) results in the following fourth order equation in k_z for TTI media

$$C_4k_z^4 + C_3k_z^3 + C_2k_z^2 + C_1k_z + C_0 = 0 \quad . \tag{4}$$

Now, rewriting eq. (4) by sorting the terms into powers of the frequency, one can show that the odd powers drop out and (4) can be written in the following polynomial form:

$$av_{P0}^4 + bv_{P0}^2\omega^2 + \omega^4 = 0 \quad , \tag{5}$$

where

$$a = (\varepsilon - \delta)(½\sin^2 2\phi k_z^4 - \sin 4\phi k_x k_z^3 - \sin^2 2\phi k_x^2 k_z^2$$

$$+ 2\cos^2 2\phi k_x^2 k_z^2 + \sin 4\phi k_x^3 k_z + ½\sin^2 2\phi k_x^4) \quad , \tag{6}$$

and

$$b = 2\varepsilon \sin 2\phi k_x k_z - (1 + 2\varepsilon \sin^2\phi)k_z^2 - (1 + 2\varepsilon \cos^2\phi)k_x^2 \quad . \tag{7}$$

The order of (5) can be reduced by substituting $x = \omega^2$, and the two pairs of roots are expressed as

$$\omega = \pm v_{P0}\{[-b \pm b\{1 - (4a/b^2)\}^{½}]/2\}^{½} \quad . \tag{8}$$

In eq. (8), considering only solutions of real frequencies and using the first order Taylor and Padé approximations of the internal square root, we obtain the following expressions to the two pairs of roots, respectively, as

$$\omega = \pm v_{p0}[(a - b^2)/b]^{½} \quad , \tag{9}$$

and

$$\omega = \pm v_{p0}[(2ab - b^3)/(b^2 - a)]^{½} \quad , \tag{10}$$

when the negative signal in the internal square root is considered.

Now, when the positive signal in the internal square root is considered, again using Taylor and Padé approximations, we obtain the following expressions:

$$\omega = \pm v_{p0}(-a/b)^{½} \quad , \tag{11}$$

and

$$\omega = \pm v_{p0}[-ab/(b^2 - a)]^{½} \quad . \tag{12}$$

For the case of elliptic anisotropy, $a = 0$, the positive signal in the internal square root will produce a null frequency result and do not reduce to the VTI case. This suggests that only the negative internal square root corresponds to the propagation of the P-wave mode.

The dispersion relations for modeling the propagation of the pure P-wave in a TTI medium correctly are given by

$$\omega^2 = v_{p0}^2[(a - b^2)/b] \quad , \quad \text{(Taylor)}, \tag{13}$$

and

$$\omega^2 = v_{p0}^2 b[(2a - b^2)/(b^2 - a)] \quad , \quad \text{(Padé)}. \tag{14}$$

A similar analysis was presented by Klíe and Toro (2001) for VTI media.

Again, using the plane wave relationships (2) and (3), we can obtain the approximated expressions for the phase velocity as:

$$v^2(\theta)/v_{p0}^2 = 1 + 2\varepsilon\sin^2\theta - 2(\varepsilon-\delta)\sin^2\theta\cos^2\theta/(1 + 2\varepsilon\sin^2\theta) \quad , \tag{15}$$

$$v^2(\theta)/v_{p0}^2 = (1 + 2\varepsilon\sin^2\theta)$$

$$\times \{1 - 2(\varepsilon-\delta)\sin^2\theta\cos^2\theta/[(1 + 2\varepsilon\sin^2\theta)^2 - 2(\varepsilon-\delta)\sin^2\theta\cos^2\theta]\} \quad . \tag{16}$$

In Figs. 1 and 2 we compare the expressions obtained by us, eqs. (15) and (16), with the phase velocity exact expression (Tsvankin, 1996), acoustic approximation of Alkhalifah (2000), the weak approximation and the acoustic approximation of Zhan et al. (2011), to different Thomsen parameters.

The expressions (13) and (14) for TTI media can be modeled with constant parameters of v_{p0}, ε, δ and ϕ. When we consider a medium with variation of the above parameters, we have to make an approximation in the denominator, so that it only depends on the wave numbers. This is done because

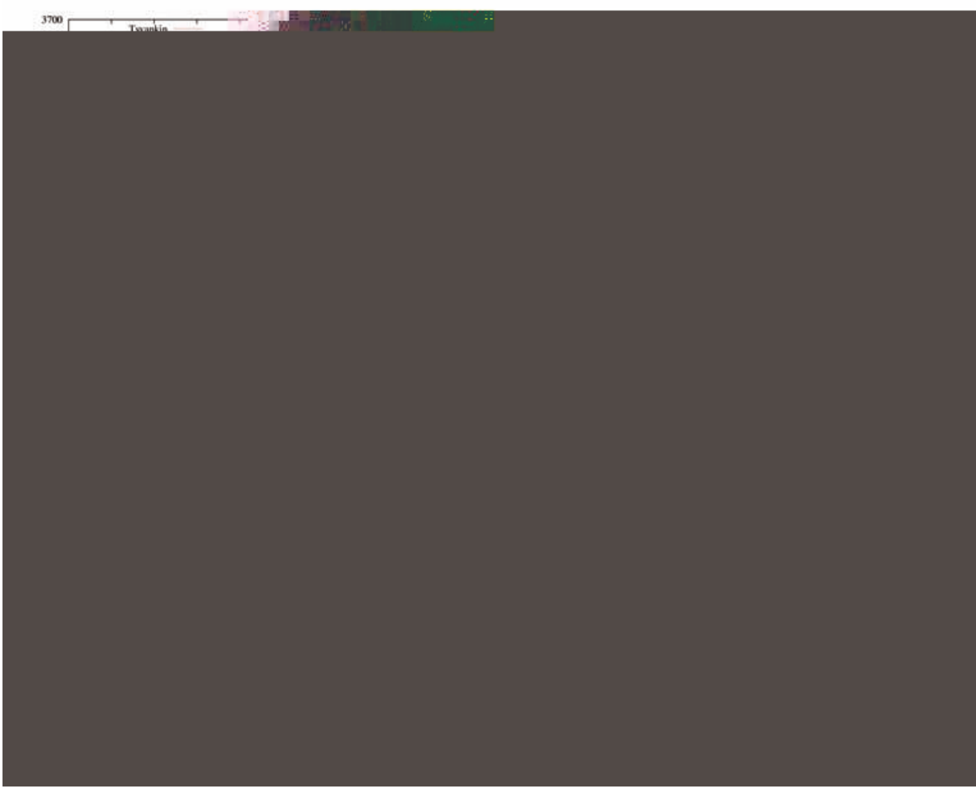

Fig. 1. Phase velocity to TTI media by Tsvankin, Pestana, Alkhalifah, for weak anisotropy and proposed approximations with symmetry axis: (a) 0°, (b) 30°, (c) 60° e (d) 90°. The anisotropy parameters are ε = 0.25 and δ = −0.075, the velocity of P- and SV-waves are 3000 m/s and 1500 m/s in parallel direction to the axis of symmetry.

it is computationally more convenient to work in the mixed Fourier-space domain in the numerator, thus making easier the numerical solution of the algorithm (Zhan et al., 2011).

If we make $\varepsilon = 0$ in the denominator of eqs. (13) and (14), we obtain eqs. (17) and (18), respectively. This approximation may cause an alteration of the amplitude without changing the shape of the wavefront (Zhan et al., 2011).

To simplify eqs. (13) and (14), we introduce the terms A, B, C, D, E, F, G, H and I, in the form:

$$A = \tfrac{1}{2}\sin^2 2\phi, \quad B = \sin 4\phi, \quad C = \sin^2 2\phi, \quad D = 2\cos^2 2\phi, \quad E = \sin 4\phi,$$

$$F = \tfrac{1}{2}\sin^2 2\phi, \quad G = 2\varepsilon\sin 2\phi, \quad H = 1 + 2\varepsilon\sin^2\phi, \quad I = 1 + 2\varepsilon\cos^2\phi.$$

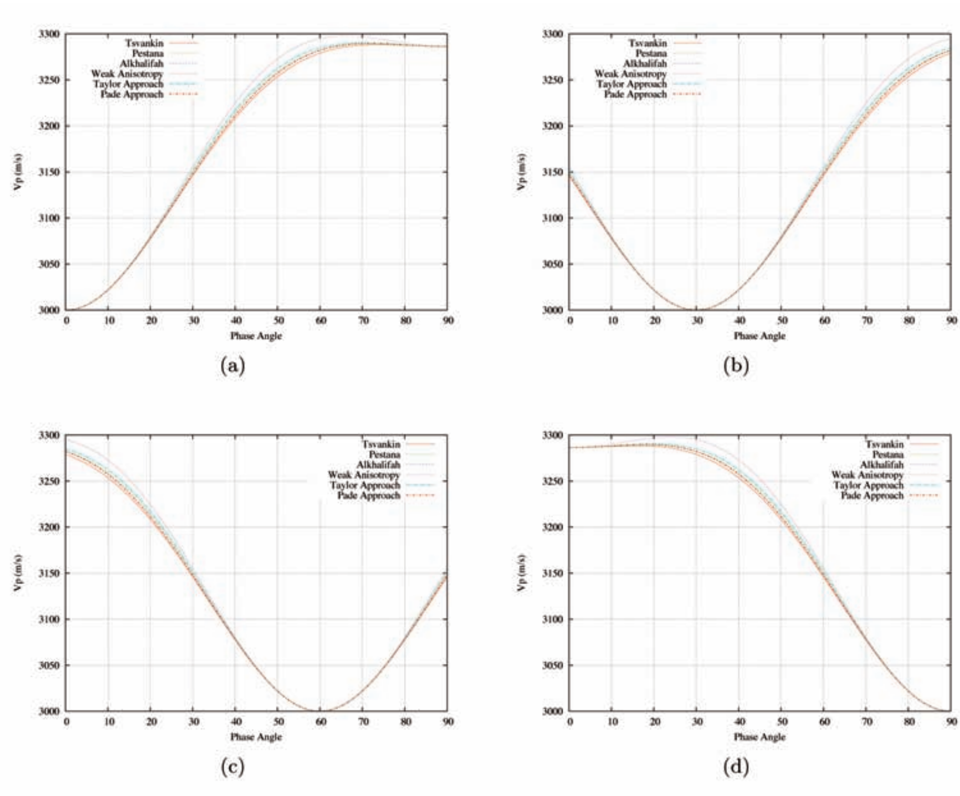

Fig. 2. Phase velocity to TTI media by Tsvankin, Pestana, Alkhalifah, for weak anisotropy and proposed approximations with symmetry axis: (a) 0°, (b) 30°, (c) 60° e (d) 90°. The anisotropy parameters are $\varepsilon = 0.1$ and $\delta = 0.25$, the velocity of P- and SV-waves are 3000 m/s and 1500 m/s in parallel direction to the axis of symmetry.

Eqs. (13) and (14), after the simplification in the denominator, are as follows:

$$\omega^2 = -v_{p0}^2\{[B(\varepsilon-\delta) - G]k_xk_z^3 + [C(\varepsilon-\delta) - D(\varepsilon-\delta) + H + I]k_x^2k_z^2$$

$$+ [H - A(\varepsilon-\delta)]k_z^4 - [E(\varepsilon-\delta) + G]k_x^3k_z + [I - F(\varepsilon-\delta)]k_x^4\}/(k_x^2 + k_z^2) \ , \qquad (17)$$

and

$$\omega^2 = -v_{p0}^2\{[G - (\varepsilon-\delta)(AG+BH)]k_xk_z^5 - [H - (\varepsilon-\delta)AH]k_z^6 - [2H + I$$

$$- (\varepsilon-\delta)\{AI+BG+H(D-C)\}]k_x^2k_z^4 + [2G - (\varepsilon-\delta)\{BI+G(D-C)-EG\}]k_x^3k_z^3$$

$$- [H + 2I + (\varepsilon-\delta)\{EG-I(D-C)-FH\}]k_x^4k_z^2 + [G - (\varepsilon-\delta)(FG-EI)]k_x^5k_z$$

$$- [I - (\varepsilon-\delta)FI]k_x^6\}/(k_z^4 + 2k_x^2k_z^2 + k_x^4) \ . \qquad (18)$$

NUMERICAL MODELING

In order to obtain the partial differential equations of second order in time for modeling and migration, the Fourier relations: $k_x \rightarrow i(\partial/\partial x)$, $k_z \rightarrow i(\partial/\partial z)$, $\omega \rightarrow -i(\partial/\partial t)$ are used in eqs. (17) and (18).

Zhan et al. (2011) showed a similar procedure, in which a set of second order differential equations in time permits the full separation of the P- and SV-wave components. Now, the second order differential equations associated to eqs. (17) and (18) are free of the diamond artifact making the seismic modeling easier to be implemented.

In this implementation the temporal derivative is solved through the Rapid Expansion Method (REM) using Chebyshev polynomial expansion (Pestana and Stoffa, 2010), and the pseudospectral method to the spatial derivatives, taking advantage of the wavenumber expressions (17) and (18), respectively.

Fig. 3 shows the wavefront obtained for a TTI homogeneous medium rotated by an angle of 45° in the tilted axis of symmetry. There we note the complete elimination of the diamond artifact.

Stability

Modeling and migration based on TTI media wave equations are generally unstable when applied to inhomogeneous media, especially in areas where the axis of symmetry changes abruptly (Zhang et al., 2011). Wave equation for TTI media using only P-waves is more stable than systems of coupled equations and is also free of S-wave artifacts. However, if we consider the coupled equations, the stability is only guaranteed for media where $\varepsilon > \delta$, where ε and δ are the Thomsen's parameters.

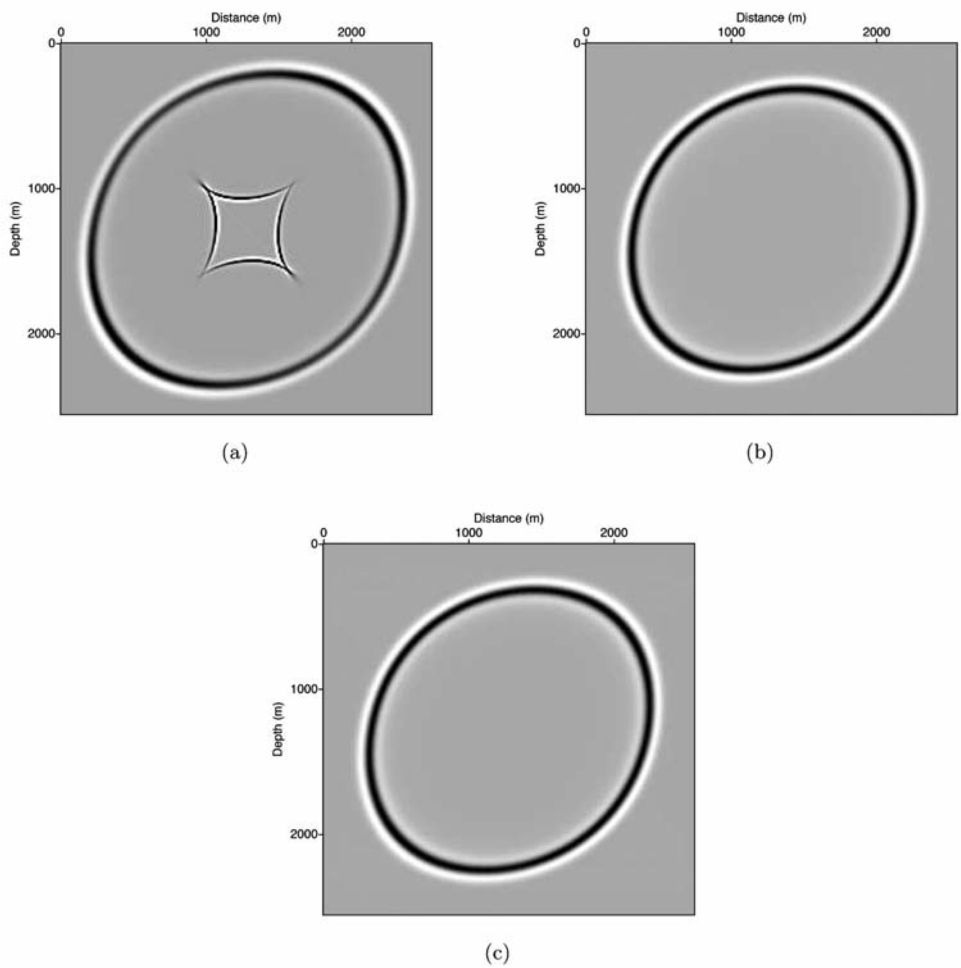

Fig. 3. P-wavefield snapshots with $\varepsilon = 0.2$, $\delta = 0.1$ and $\phi = 45°$ for the cases: (a) TTI with the effect to SV-wave using Zhou et al. (2006) eqs., (b) TTI with eq. (10) and (c) TTI with eq. (11).

Some instability problem result from the computation of mixed spatial derivatives due to the strong contrasts of anisotropic parameters, particularly the axis of symmetry (ϕ). This problem also appears in the solution of the P-wave equation and seems to be the main cause of the instability of the algorithms used to solve the new P-wave equations for TTI media (Zhan et al., 2012).

Yoon et al. (2010) suggested to compute the gradient of ϕ and set $\varepsilon = \delta$ to the locations with high gradient values in order to stabilize the solution of the proposed coupled equations for TTI media. Zhang et al. (2011) introduced a differential operator with rotated coordinates also aiming the stabilization of the

TTI coupled equations of Zhou et al. (2006). Zhan et al. (2012) proposed the application of a band-pass filter in the wave number domain to smooth the wavefield for high number in order to improve the computation of the mixed spatial derivatives and avoid the instability issue related to pure P-wave equation for TTI media.

To test the stability of pure P-wave equations compared to coupled equations, we take a model similar to Duveneck and Bakker (2011). This model has strong contrasts of Thomsen's parameters and of the axis of symmetry (ϕ), as seen in Fig. 4. When we get the impulse response of the coupled equations from the model showed in Fig. 5, we notice that in the early stages of the propagation of the wave the behaviour is stable and in the final stages it starts to become unstable. The reason for this is that the energy of the SV-wave continues to reverberate between the layers of the model.

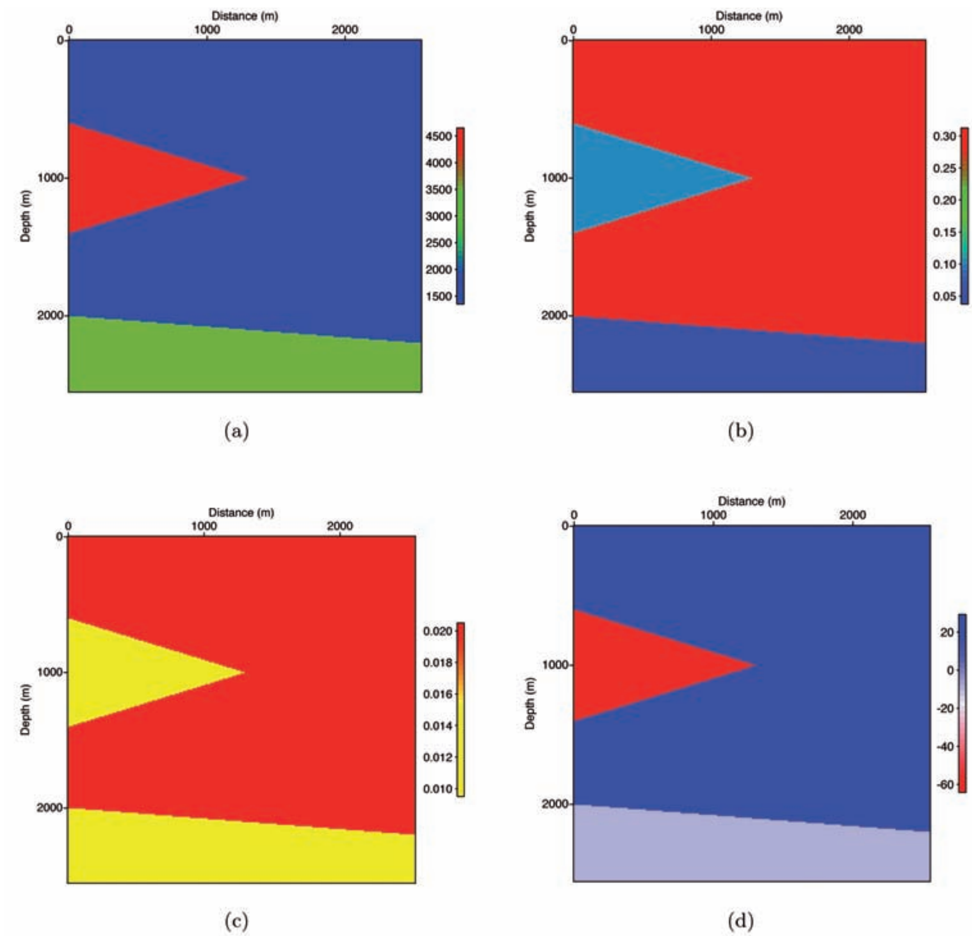

Fig. 4. Two-layer model and a wedge based on Duveneck and Bakker (2011). (a) velocity field (m/s), (b) ε field, (c) δ field and (d) ϕ field (angle in degrees).

Using eq. (17), we model the impulse response of the P-wave in the wedge model of Fig. 4, as seen in Fig. 6. We notice in Fig. 6, that the wave propagation in every stage of time is stable and does not have the presence of the SV-wave, demonstrating that the pure P-wave equations are more stable than the coupled equations.

In the case of migration of data sets with more complexity than the wedge model, we have more factors that can make the algorithm unstable; factors such as the presence of mixed derivatives and strong contrasts of anisotropic parameters, particularly the axis of symmetry. To overcome this instability problem, we propose to apply a 2D moving average smoothing filter to the parameter ϕ, in order to stabilize the computer implementation of pure P-waves equations for TTI media proposed in this work. After several trials with different values for the smoothing filter, we found that a length of 10 × 10 grid

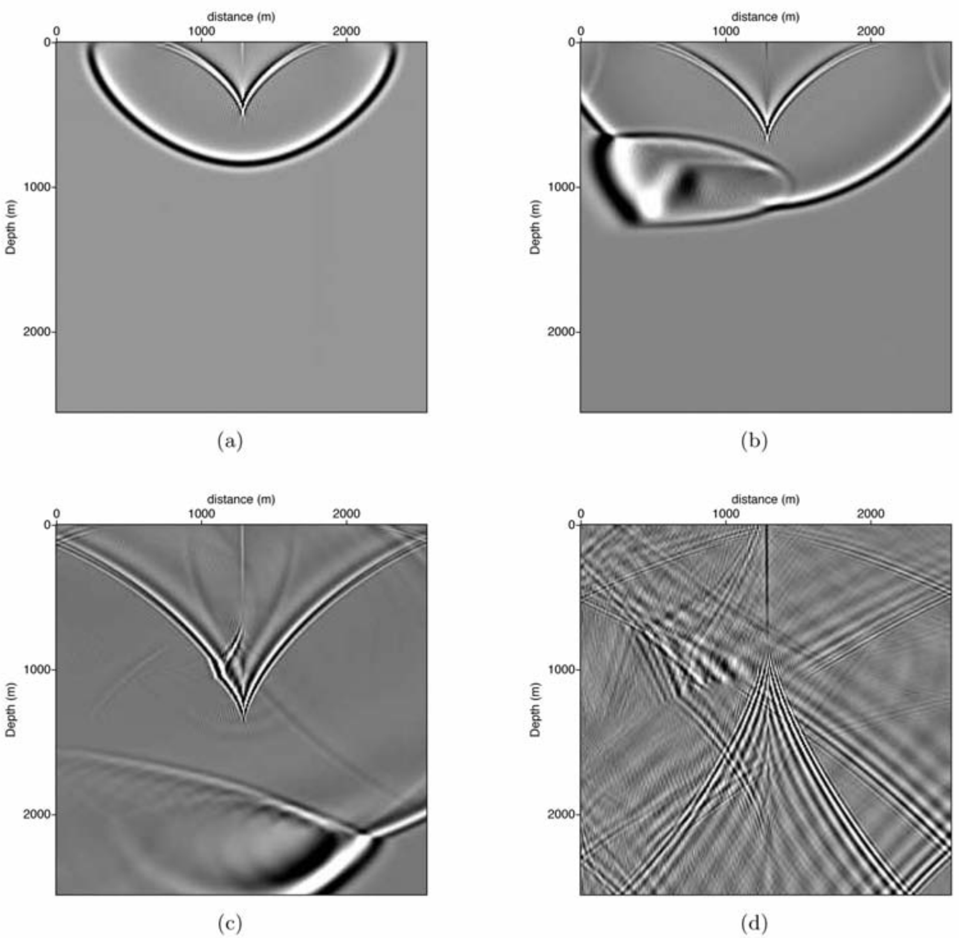

Fig. 5. Wave field snapshots of the wedge model with the TTI coupled wave equation of Zhou et al. (2006) for the cases: (a) t = 0.6 s, (b) t = 0.8 s, (c) t = 1.6 s and (d) t = 5.4 s.

points for the smoothing filter was enough to migrate the part two of the BP-TTI dataset.

RTM IN TTI MEDIA

The technique of reverse time migration (RTM) consists of moving the reflectors to a more accurate position, based on the solution of the full wave equation. The implementation of the RTM can be done through the solution of the wave equation using the finite difference method to approximate the spatial and temporal derivatives. Pestana and Stoffa (2010) proposed another technique for the solution of the RTM called rapid expansion method (REM) that consists of obtaining a recursive solution of the wave equation using a series expansion with Chebyshev polynomials (Tal-Ezer et al., 1987) and using pseudospectral

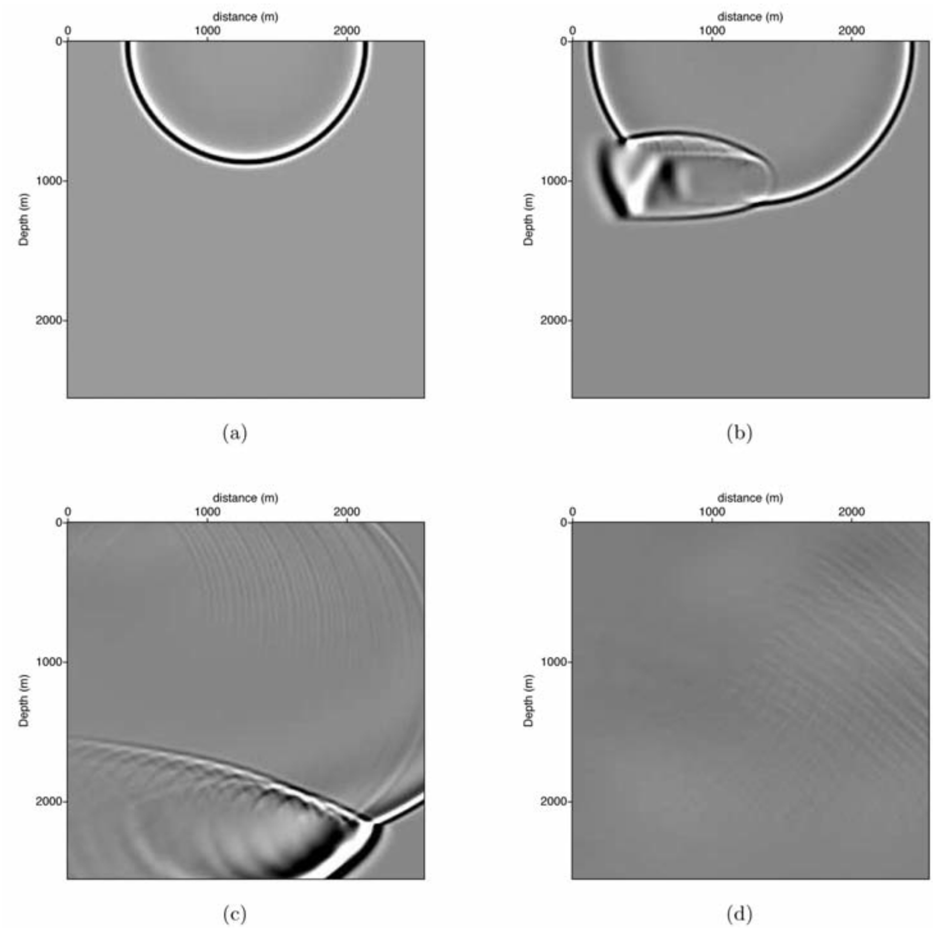

Fig. 6. Wave field snapshots of the wedge model for TTI media of the pure P- wave equation (10), in the cases: (a) t = 0.6 s, (b) t = 0.8 s, (c) t = 1.6 s and (d) t = 5.4 s.

methods to compute the spatial derivatives. Through the REM, the wave field can be calculated from the recorded wave field for any time of propagation and for any order of approximation for the time derivative.

The BP-TTI model was migrated using RTM with the technique REM. The synthetic BP-TTI dataset corresponds to a TTI model with strong lateral variations of v_{p0}, ε, δ and ϕ parameters, as seen in Fig. 7. Fig. 9(a) shows the parameter ϕ with a smooth filter of 10 × 10 grid points.

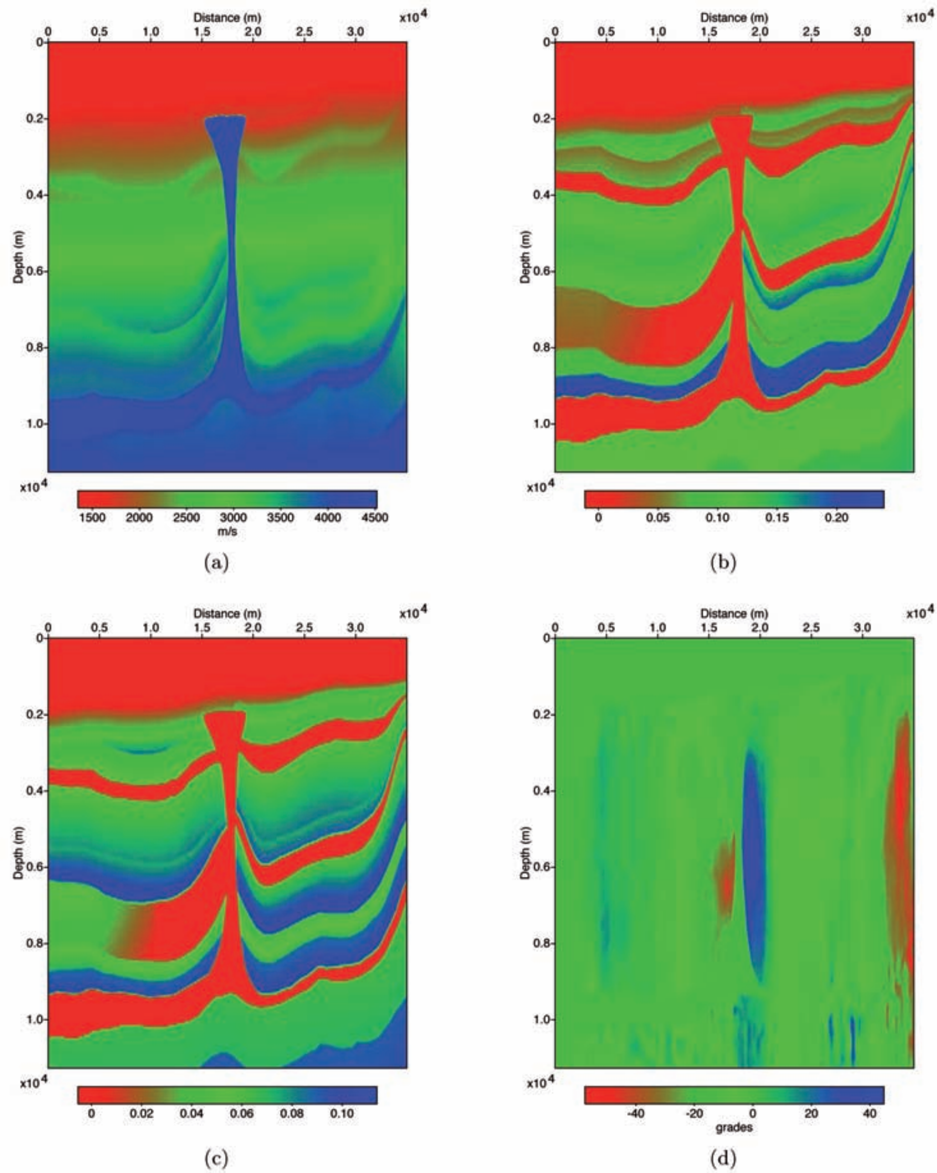

Fig. 7. Parameters of the BP-TTI model, (a) v_{p0}, (b) ε, (c) δ and (d) ϕ.

In Fig. 8(a) we can see the isotropic migration of the BP-TTI dataset. In this result it is not possible to delineate the edges of the salt dome, the lateral continuity of some reflectors are not very clear and by not considering the anisotropic media, we know that the reflectors may be poorly positioned. In Fig. 8(b), we show the VTI migration of BP-TTI dataset. In this picture we have an improvement in the lateral continuity of the reflectors and the limits of the salt dome are better defined than in Fig. 8(a), thus facilitating their spatial delimitation in the section.

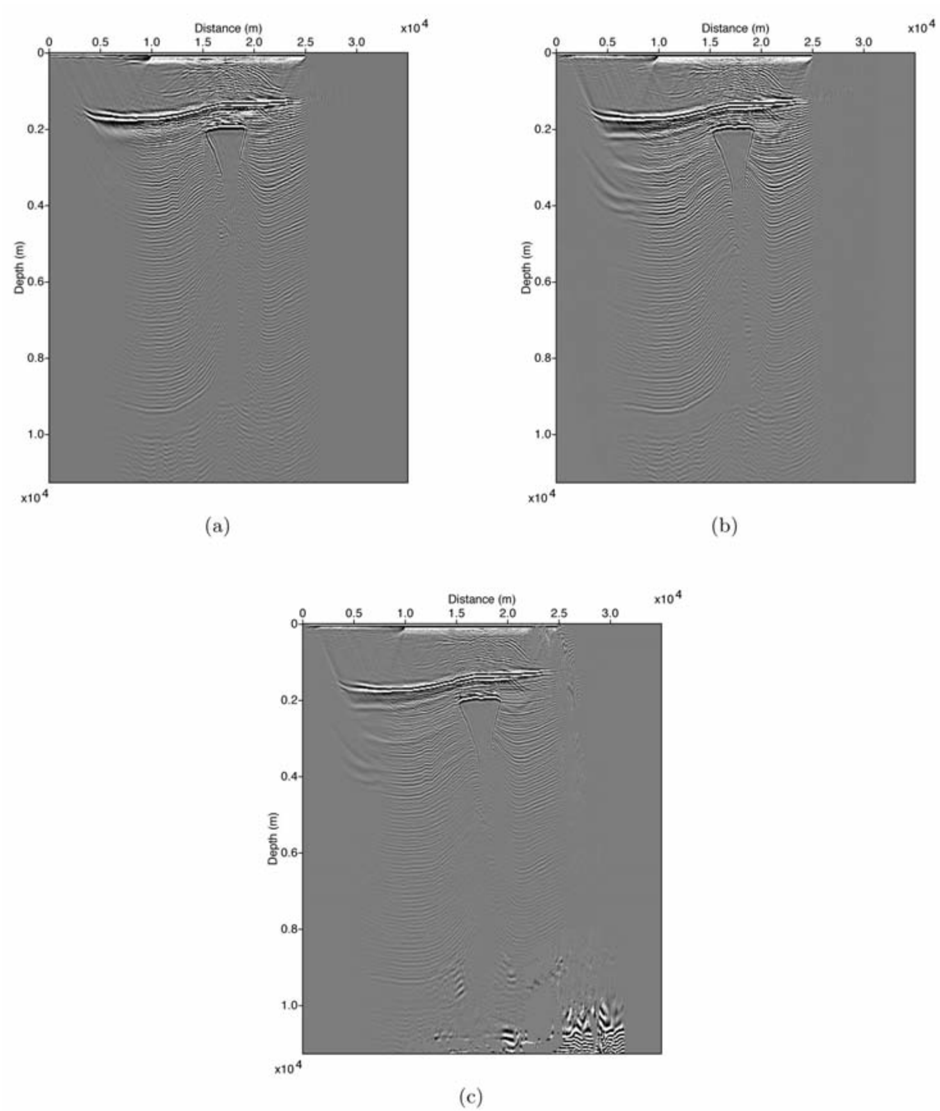

Fig. 8. Migration of the BP-TTI model in the cases: (a) isotropic, (b) VTI ($\phi = 0$) and (c) without smoothing in the parameter ϕ.

In Fig. 8(c), we can see the TTI migration of the BP-TTI dataset, where we notice that the migration presents a lot of noise, masking the majority of the reflectors, especially in the lower section making it difficult the interpretation of the seismic image. The application of a smoothing filter of 10 × 10 points, avoids the noise in the migrated section, as seen in Fig. 9(b) and 9(c). We can see that continuity of reflectors and the edge of the salt dome are clear and continuous, resulting in an optimal section to be interpreted in a stratigraphic and structural way.

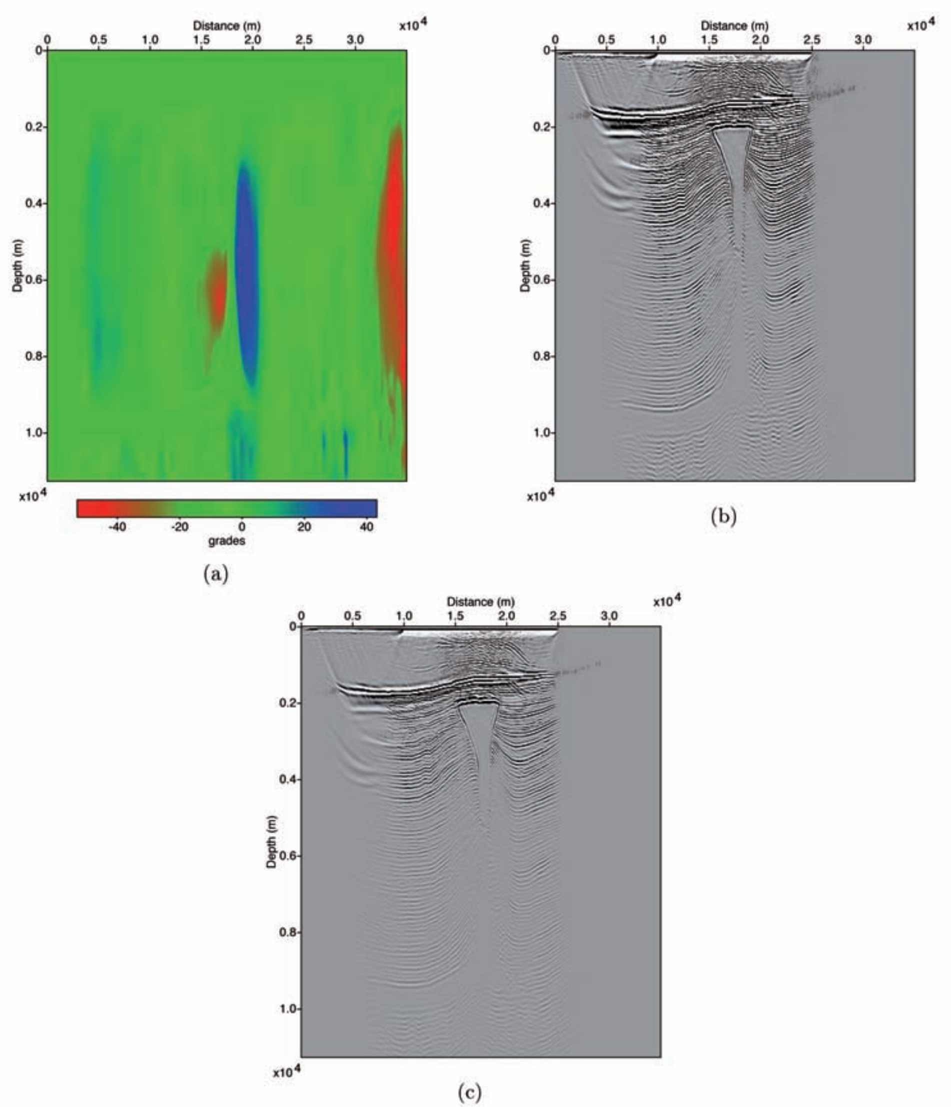

Fig. 9. (a) parameter of anisotropy ϕ with length smoothing filter of 10 x 10 grid points. Migrations of the BP-TTI using given in (a). In (b) approximation with Taylor series [eq. (17)] and (c) approximation with Padé series [eq. (18)].

CONCLUSIONS

We present two new equations for modeling and migrating P-wave in TTI media. These new equations have the advantage of being of second order derivative in the time and they are numerically more stable compared to the coupled equations. These equations can also completely separate for P- and SV-waves.

Observing the phase velocity curves, we conclude that the equations proposed in this paper have good approximations to the exact equation compared to the equations of Zhan et al. (2011), Alkhalifah (2000) and the first order approximation of weak anisotropy.

The numerical stability in the TTI migration algorithm was achieved through the smoothing parameter ϕ in the seismic section. In the case of the second part of the model BP-TTI, we successfully use a smoothing filter of 10 $\times$ 10 grid points applied to the field ϕ, thus obtaining a good quality migrated section.

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